Loss Aversion, Inefficiency and Policy Interventions*

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August 2019

Abstract

The paper studies a production economy with exotic preferences featuring loss aversion, a core concept commonly accepted in behavioral economics. The representative household obtains utility directly from fluctuations in asset returns, in addition to consumption. I uncover that the competitive equilibrium is inefficient as long as the agent is loss averse without any frictions, due to pecuniary externalities. The household does not internalize the price effect on her welfare so that she invests more in capital than the optimum requires. The policy to implement the constrained optimal allocations rationalizes policy interventions: the government should tax capital to reduce capital stock and raise the utility from asset returns.

JEL classification: E71, D62, E62

Keywords: loss aversion, fluctuations, inefficiency, pecuniary externalities, capital taxation

*I am grateful to Salvador Ortigueira, Andrés Erosa, Matthias Kredler, Pedro Gomes, Hernán Seoane, Manuel Santos, Anastasios Karantounias, Karl Walentin, Martín Gonzalez Eiras, Stefan Niemann, Omar Rachedi and Alessandro Perri for comments. I give thanks to the organizers of student seminars in UC3M and the ENTER network.

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1 Introduction

Acemoglu and Scott (1997) and Van Nieuwerburgh and Veldkamp (2006) among others have suggested asymmetric business cycles. A standard general equilibrium model finds it difficult to reflect this phenomenon. Such a model may misunderstand agents' behaviors over the business cycles, especially in the recessions, and lead to an inappropriate policy prescription. Therefore, this paper explores whether a model generating the asymmetry with loss aversion behaves differently in competitive equilibrium. Based on the model, I investigate the efficiency of the competitive equilibrium and the corresponding policy to achieve optimal allocations.

Kahneman and Tversky (1979) propose loss aversion as a part of prospect theory. Loss aversion postulates that economic agents evaluate decisions based on a reference point, which implies that utility can be generated from not only the absolute value, but also a change in value. Besides, economic agents value gains differently from the way in which they value losses as experimental evidence shows. They obtain a greater disutility from a loss than a utility from the same amount of gain. The constructed asymmetry in the preferences is a natural candidate for the asymmetry over the business cycles.

Loss aversion has been confirmed not only from experiments, but also by empirical studies. For instance, Pope and Schweitzer (2011) test for the presence of loss aversion using a large sample of professional golfers' performance on the PGA Tour. They verify that even the best golfers show loss aversion. Camerer et al. (1997) use data on cabdrivers in New York City to reveal that drivers are afraid of falling below a target income, consistent with loss aversion. The driver decides working hours of a day largely depending on the comparison between actual daily income and the target: the driver stops sooner if he earns the target income more quickly; furthermore, earning less than the target has a stronger effect than earning the equivalent amount more than the target. Rosenblatt-Wisch (2008) finds evidence of loss aversion in aggregate consumption. Foellmi et al. (2018) find that loss aversion prevails at the aggregate level in all OECD countries they study, correlated with economic fundamentals such as the level of GDP and consumption per capita.

Loss aversion is often applied in finance. For example, Benartzi and Thaler (1995) apply loss aversion to explain the equity premium puzzle. Focusing on certain asset markets, they claim that a reasonable loss aversion degree generates a high equity premium if agents check their account once a year. Ang et al. (2006) show that agents place greater weight on downside risk, indicating loss aversion in individual investors. Dimmock and Kouwenberg (2010), and

O'Connell and Teo (2009) report that individual investors and large institutional investors all exhibit loss aversion by matching investment behaviors with prospect theory.

General equilibrium theories, nonetheless, rarely consider loss aversion. I embed loss aversion in a business cycle model. Besides consumption, a loss averse household obtains utility from expected gains from risky assets relative to a certain reference point. I characterize the competitive equilibrium and point out the distinction of the equilibrium.

I show analytically that the competitive equilibrium is inefficient by considering a constrained optimality problem following Davila et al. (2012). The atomistic household takes prices as given and does not internalize the influence of her choice on prices. With exotic preferences, prices directly affect the utility so that a pecuniary externality creates a gap between the equilibrium and the constrained optimum even without any idiosyncratic risks or frictions.

In a special case where the gain-loss utility is linear with a kink, I discuss analytically how capital stock in equilibrium evolves with the loss aversion degree indicating how much a loss affects welfare relative to a gain, and with the relative weight of the gain-loss utility showing the level of concern over the direct impact of fluctuations in asset prices on welfare. I show that the more loss aversion and the more concern over the gain-loss utility, the less investment in risky assets in equilibrium given the current state and other parameters. Given a path of aggregate productivity, the initial condition of capital stock, and other parameters, an increase in the loss aversion degree reduces investment in risky assets in all periods.

Applying quantitative analysis, I show the asymmetric impacts of positive shocks and negative shocks. I present the business cycle statistics of the competitive equilibrium and point out how loss aversion improves the behavior of some variables in the business cycle model. I compute the constrained optimum and the competitive equilibrium to confirm that the equilibrium deviates from the optimum. With baseline calibration, the welfare loss from the constrained optimum to the equilibrium reaches 0.33%, measured by consumption equivalent variation. The gap between the equilibrium and the constrained optimum also mirrors the cost of fluctuations over the business cycle in my model, which is much larger than many previous studies and not "negligible" as claimed by Lucas (1987). This is because the household directly experiences disutility from potential losses from investment in addition to more volatile consumption.

I examine whether policy interventions can correct the inefficiency of competitive equilibrium. I add into the model a government sector which levies capital income taxes and rebates all the tax revenues by lump-sum subsidies. The government attempts to implement the constrained optimal allocations by these policy instruments. Judd (1985) and Chamley (1986) demonstrate that the optimal capital income tax rate tends to zero in the long run, which has been confirmed when relaxing a number of assumptions. Chari et al. (1994) study the optimal policy over the business cycle quantitatively and show that the long-run mean of capital tax rate is close to zero even with a relatively high risk aversion. In my model, the optimal policy requires a high capital income tax rate(more than 18%) with baseline calibration of loss aversion parameters. In comparison, the optimal capital income tax rate is always zero if the feature of loss aversion is removed.

In an augmented model with two assets, I apply the numerical analysis to confirm that the inefficiency of the competitive equilibrium holds if relaxing the assumption of a constant reference point of loss aversion. The gap between the constrained optimum and the market equilibrium even increases to above 13% when the model generates a realistic equity premium in equilibrium.

This paper belongs to a branch of macroeconomic research that considers exotic preferences. Many papers apply Epstein-Zin preferences to either separate intertemporal substitution from risk aversion or obtain a relatively high equity premium. Angeletos and Calvet (2006) and Angeletos (2007) both apply Epstein-Zin preferences to see the effect of idiosyncratic production risks on the equilibrium over the business cycle and on economic growth. Epstein-Zin preferences, by differentiating the elasticity of intertemporal substituion from risk aversion, help to identify that the underlying factor lies in the elasticity of intertemporal substitution. Croce et al. (2012) investigate the optimal fiscal policy which functions through the channel of asset prices. They also use Epstein-Zin preferences to generate a realistic equity premium. Croce (2014) generates a high equity premium with Epstein-Zin preferences and long-run risks. Karantounias (forthcoming) designs optimal fiscal policy with recursive utility. The paper finds that the planner should tax less in bad times and more in good times, and that optimal policy calls for an even stronger use of debt returns as a fiscal absorber. Another major group applies habit formation. Constantinides (1990) shows that habit persistence resolves the equity premium puzzle. Otrok et al. (2002) explore the nature of this resolution and find that the equity premium in the habit model is driven by high-frequency volatility, which is generally incompatible with the smooth characteristics of U.S. consumption. Chugh (2007) derives Ramsey fiscal and monetary policies with habit formation. Habit persistence partly predicts highly persistent inflation. Although other research on exotic preferences is equally intriguing and educative, I have decided not to mention them due to limited space. The preferences in my paper, instead of the aforementioned relatively common ones, include a gain-loss utility that features loss aversion. In particular, the paper generates the asymmetry over the business cycles whose property is difficult to be reflected in models with other exotic preferences. Then the paper studies how loss aversion affects the market equilibrium and its efficiency.

The paper is closer to the growing literature that uses loss aversion in the research of general equilibrium. Barberis et al. (2001) study asset pricing considering loss aversion in financial wealth and discover that their framework can explain the high mean, excess volatility and predictability of stock returns. Barberis and Huang (2001), and Berkelaar and Kouwenberg (2009) explore equilibrium firm-level stock returns with loss aversion in two different economies. Andries (2015), De Giorgi and Legg (2012) and Pagel (2016) readdress asset pricing with loss aversion. Easley and Yang (2015) explore whether loss averse investors survive in the market and affect the long-run asset prices. Pagel (2017) uses the expectation-based reference-dependent preference featuring loss aversion to explain empirical observations about life-cycle consumption. Ahrens et al. (2017) develop a theory under loss aversion which successfully explains why prices are more sluggish upwards than downwards in response to temporary demand shocks, while they are more sluggish downwards than upwards in response to permanent demand shocks as empirical evidence finds. Lepetyuk and Stoltenberg (2013) reconcile the changes in consumption inequality in the data in response to an increase in income inequality with loss aversion preferences. Yet all these papers assume an endowment economy. On the contrary, I investigate a production economy in which the model is able to discuss the effect of loss aversion on the supply of capital and its consequent effect on production, consumption and welfare.

My paper is closest to a few studies that apply loss aversion in a general equilibrium model within a production economy. Grüne and Semmler (2008) extend the asset pricing model with loss aversion proposed by Barberis et al. (2001) in a production economy. Chen (2013) quantitatively evaluates the theory of Loss Aversion/Narrow Framing as a resolution to the Equity Premium Puzzle and concludes that the theory is unable to jointly describe the equity premium and labor's elasticity of supply. Santoro et al. (2014) develop a general equilibrium model with loss aversion to rationalize that monetary policy exerts asymmetric effects on output over expansions and depressions. Chen (2015) accounts for asymmetric business cycles with loss aversion. My research differs from theirs in research topics: I focus on the inefficiency of the market equilibrium and related fiscal policies to implement the constrained efficient allocations. Thus I consider that my paper is a novel attempt from a theoretical perspective.

My paper is in line with the literature that discusses constrained inefficiency, particularly with pecuniary externalities. Gromb and Vayanos (2002), and Lorenzoni (2008) point out that pecuniary externalities can lead to inefficiency. The works of Bianchi (2011), Davila et al. (2012), Benigno et al. (2016), Farhi and Werning (2016), Gersbach and Rochet (2017) and Dávila and Korinek (2018) develop the literature in the directions of theoretical exploration or policy issues. Nonetheless, all these studies generate inefficiency through the channel of real frictions. My model, unlike the above works considering frictions, proves that only exotic preferences featuring loss aversion are enough to result in inefficiency.

Finally, my paper is related to a large amount of papers involving the discussion of whether the government should tax capital. Judd (1985) and Chamley (1986) demonstrate that the optimal capital income tax rate tends to zero in the long run, which has been confirmed when relaxing a number of assumptions. Chari et al. (1994) study the optimal policy in the business cycle quantitatively and show that the long-run mean of the capital tax rate is close to zero even with relatively high risk aversion. Aiyagari (1995), Conesa et al. (2009) and Panousi and Reis (2012), among others, suggest that the government should tax capital even in the long-run in a general equilibrium model with idiosyncratic risks. The result of my paper favors the latter, but from a different channel. It indicates that the government should tax capital because a higher return to capital raises the gain-loss utility and heightens welfare.

The paper is organized as follows. I develop a parsimonious production economy model with loss aversion in Section 2 and determine the competitive equilibrium. In Section 3, I show the inefficiency of the competitive equilibrium and characterize the constrained optimum. Section 4 studies a special case where the gain-loss utility is linear with a kink. I also analyze the comparative statics of the loss aversion components. Section 5 is devoted to the numerical analysis. Section 6 formulates a model with a government and develops a policy to implement the constrained optimal allocations. Section 7 develops an augmented model and discusses the efficiency of equilibrium in this augmented model. Section 8 concludes the study.

2 The Model with Loss Aversion

This section presents a parsimonious real business cycle model with preferences in consumption and shifts in asset returns. The latter features loss aversion. I assume that only risky assets, capital, is traded in this and in the following sections to obtain some analytical results. Section 7 will relax the assumption and consider the portfolio choice. I characterize the competitive equilibrium.

2.1 Economy

The economy is populated by a continuum of identical households, each of whom is endowed with one unit of time in each period. The households maximize expected lifetime utility,

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U_t,\ \beta \in (0,1). \tag{1}$$

A representative firm produces a single consumption good with labor, n_t, and capital, k_t. The total output, Y_t, is consumed or used to augment the capital stock. The feasibility constraint is

$$c_t + k_{t+1} = Y_t + (1-\delta)k_t, \tag{2}$$

where $\delta \in (0,1)$ is the depreciation rate.

Product and factor markets are assumed to be competitive.

2.2 Firm

The firm takes as given the wage rate, w_t, and the rental rate, r_t, hires labor and rents capital from the households, produces final consumption goods and maximizes its profit,

$$\Pi_t = Y_t - r_t k_t - w_t n_{t.} \tag{3}$$

I assume that the production function, $Y_t = Z_t F(k_t, n_t)$, has constant returns to scale and that it is strictly increasing and strictly concave in each argument. Exogenous aggregate productivity, Z_t, follows an AR(1) process,

$$\ln Z_{t+1} = \rho_z \ln Z_t + \sigma_z \epsilon^z_{t+1}, \tag{4}$$

where ρ_z denotes the autoregressive parameter for the moving process of productivity, σ_z represents the standard deviation of one-time technological innovation, and innovation, ϵ^z_{t+1}, is independently distributed as a standard normal for any $t \geq 0$.

2.3 Households

Each household is endowed with some initial capital, k_0. At period t, a household receives income from labor supply and capital rental, and then determines the amount of consumption and capital accumulation by maximizing (1) subject to the following sequences of budget constraints and nonnegativity constraints:

$$c_t + k_{t+1} = w_t n_t + r_t k_t + (1-\delta)k_t,\ \forall t, \tag{5}$$

$$c_t \geq 0,\ k_t \geq 0,\ 0 \leq n_t \leq 1,\ \forall t. \tag{6}$$

The uncertainty of the return to capital rental arises from unknown productivity shocks.

2.4 Preference Specification

As the main feature of this paper, I assume that a household directly enhances her utility, in addition to consumption, if she expects a gain in investment. This assumption implies that the household cares about fluctuations in investment markets independent of total wealth. It reflects the observation that individuals in reality feel excited when they succeed in the capital market. Mathematically, I express the instantaneous utility at t consisting of consumption in the current period, c_t, and expected gains from capital investment in the next period with respect to a reference point defined later, X_{t+1}, as

$$U_t = u(c_t) + \eta_t \beta \mathbb{E}_t \left[v(X_{t+1}) \right], \tag{7}$$

where u is strictly increasing, strictly concave and two times continuously differentiable in c, and v represents a gain-loss utility. η_t denotes the time-varying relative weight on utility from expected gains compared to consumption. The preferences return to standard ones merely containing consumption when $\eta_t \equiv 0$. I formulate preferences over consumption and the expected gains of capital investment in the spirit of Barberis et al. (2001), whose preference specification consists of two additively separable terms: utility from consumption and from expected one-period-after fluctuations in financial asset values. I also consider the scenario when the agent obtains the loss aversion utility from realized gains, that is, $\eta_t v(X_t)$. The corresponding first-order conditions show that the timing alternative does not affect investment decisions.

I apply the utility function over gains and losses defined by Tversky and Kahneman (1992):

$$v(X_{t+1}) = \begin{cases} X^\theta, & \text{if } X_{t+1} \geq 0; \\ -\lambda(-X)^\theta, & \text{if } X_{t+1} < 0. \end{cases} \tag{8}$$

$v(X)$ captures a central idea of prospect theory by Kahneman and Tversky (1979) that people are loss averse over changes in financial wealth. Some recent studies such as Pagel (2016) and Pagel (2017) model loss aversion with the expected consumption as the reference point. My paper, instead, targets financial wealth rather than consumption in the preference component of loss aversion. The parameter, λ, denotes the loss aversion degree and it is assumed to be strictly larger than 1, indicating that a certain amount of loss has a greater impact in absolute value than the same amount of gain. $\theta \geq 1$ measures the curvature of the S-shape of a gain-loss utility, in line with the finding of behavioral economics that agents tend to avoid risks in the gain region while seek risks in the loss region.

I define the gross returns to capital rental as $R_t = r_t + 1 - \delta$. In this paper, X_{t+1} is assumed to have the form:

$$X_{t+1} = k_{t+1}R_{t+1} - k_{t+1}\bar{R}, \tag{9}$$

where $\bar{R}$ represents a targeting return the household sets, such as the mean return.

At $t+1$, the household receives $k_{t+1}R_{t+1}$ from investment in risky assets. The household regards the mean return to capital as the reference point. Suppose that the household has already invested k_{t+1} in risky assets and will earn $k_{t+1}R_{t+1}$ in the next period; she, however, would compare the return with the average level. If $R_{t+1} > \bar{R}$, it is defined as a gain; and if $R_{t+1} < \bar{R}$, a loss. Denote $D_{t+1} = R_{t+1} - \bar{R}$, so I can rewrite $X_{t+1} = D_{t+1} \cdot k_{t+1}$. Then,

$$v(X_{t+1}) = v(D_{t+1}) \cdot k_{t+1}^\theta. \tag{10}$$

At t, the agent only has one unknown, $t+1$'s productivity, Z_{t+1}. Since the distribution of innovation is common knowledge, the agent computes the next period's expected gain and subsequent utility from it conditional on current information.

2.5 Competitive Equilibrium

I define a competitive equilibrium as a stochastic sequence of prices $\{w_t, r_t\}_{t=0}^{\infty}$, a stochastic sequence of allocations $\{c_t, k_{t+1}\}_{t=0}^{\infty}$, such that

(1) Given prices $\{w_t, r_t\}_{t=0}^{\infty}$, the household maximizes her lifetime utility by choosing $\{c_t, k_{t+1}\}_{t=0}^{\infty}$, and the firm maximizes its profit by choosing the amount of inputs.

(2) Goods market clearing: feasibility constraint, (2), holds.

In equilibrium labor supply is inelastic, $n_t = 1$. Factor prices are determined by solving the representative firm's problem,

$$r_t = Z_t F_k(k_t, 1), \tag{11}$$

$$w_t = Z_t F_n(k_t, 1). \tag{12}$$

The Euler equation characterizes the solution to the household's maximization problem:

$$u'(c_t) = \beta \mathbb{E}_t \left[R_{t+1} u'(c_{t+1}) \right] + \eta_t \beta \theta k_{t+1}^{\theta-1} \mathbb{E}_t \left[v(D_{t+1}) \right], \tag{13}$$

Using equilibrium conditions, I rewrite the gross return to capital in period $t+1$, $R_{t+1} = Z_{t+1} F_k(k_{t+1}, 1) + 1 - \delta$. By construction, only Z remains unknown in period t. If we focus on the gain-loss utility, the individual feels indifferent from obtaining the real return, R_{t+1}, and the referred average return, $\bar{R}$, when the realized value of Z_{t+1}, denoted by z_{t+1}, equals $z_{t+1}^{idf} = \frac{\bar{R}-1+\delta}{F_k(k_{t+1},1)}$. z_{t+1}^{idf} will be larger if the agent accumulates more capital. Only if the realization of the next period's productivity surpasses this cutoff can the agent obtain a positive gain-loss utility from her investment. Thus, a larger cutoff decreases the gain possibility, producing pessimistic beliefs about the expected payoff. It implies that more capital investment leads to more disutility from fluctuations in asset values given the same expected productivity.

With the above definition, I rewrite the expected gain-loss utility from $t+1$'s asset value conditional on t's information as

$$\begin{aligned} & \mathbb{E}_t \left[v\left(X_{t+1} \right) \right] \\ = & \, k_{t+1} E_t \left[v\left(D_{t+1} \right) \right] \\ = & \, k_{t+1} \left[\int_0^{z_{t+1}^{idf}} -\lambda \left(\bar{R} - R_{t+1} \right)^{\theta} \mathrm{d}\mathcal{F}_{Z_{t+1}|Z_t=z_t}(z) + \int_{z_{t+1}^{idf}}^{\infty} \left(R_{t+1} - \bar{R} \right)^{\theta} \mathrm{d}\mathcal{F}_{Z_{t+1}|Z_t=z_t}(z) \right], \end{aligned} \tag{14}$$

where $\mathcal{F}_{Z_{t+1}|Z_t=z_t}(z)$ is the conditional cumulative distribution function of the next period's productivity, Z_{t+1}, in period t. Simply put, I separate the utility from expected gains from that from expected losses and calculate each conditional expectation. The formulation is motivated by Kőszegi and Rabin (2009), where the reference point of unknown future consumption is defined as continually updated conditional expectations of future consumption in a dynamic environment. Hence this modelling approach can be viewed as a special case of their more general setting, in the sense that in a certain period, the reference point is fixed, while their model embraces both uncertain realizations and uncertain reference points.

3 Inefficient Competitive Equilibrium

This section discusses the efficiency of competitive equilibrium. I first construct a social planner's problem in which the planner is only allowed to assign capital stock in the spirit of Davila et al. (2012). I characterize the constrained optimum, the solution to the social planner's problem, and compare it with the competitive equilibrium. It indicates that in this simple model the competitive equilibrium is inefficient when the household is loss averse.

I consider a constrained efficient social planner who only assigns capital stock for the representative agent facing identical preferences. I denote the total welfare across infinite periods as W. I define the constrained efficiency of an allocation as follows:

Definition. *The allocation, $\{k_{t+1}\}_{t=0}^{\infty}$, is constrained efficient if it is feasible and if there is no other feasible allocation, $\{k'_{t+1}\}_{t=0}^{\infty}$, such that $W\left(\{k'_{t+1}\}_{t=0}^{\infty}\right) > W\left(\{k_{t+1}\}_{t=0}^{\infty}\right)$.*

The necessary condition of constrained efficiency satisfies that $\dfrac{\mathrm{d}W}{\mathrm{d}k_{t+1}} = 0$, $\forall t \geq 0$. The key issue in constructing the social planner's problem is how to determine asset prices because they appear in the utility function. The social planner is restricted to following the pricing rule that factor prices are determined by market equilibrium conditions in line with Bianchi and Mendoza (2018). The constrained optimum is the solution to

$$\max_{c_t, k_{t+1}} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ u(c_t) + \eta \beta k_{t+1}^{\theta} \mathbb{E}_t \left[v(D_{t+1}) \right] \right\}$$

subject to

$$c_t + k_{t+1} = Z_t F(k_t, 1) + (1-\delta) k_t, \; \forall t,$$

where

$$D_{t+1} = Z_{t+1} F_k(k_{t+1}, 1) + 1 - \delta - \bar{R}.$$

The first-order condition for the social planner's problem is

$$u'(c_t) = \beta\mathbb{E}_t\left[R_{t+1}u'(c_{t+1})\right] + \eta_t\beta\theta k_{t+1}^{\theta-1}\mathbb{E}_t\left[v(D_{t+1})\right] + \eta_t\beta k_{t+1}^{\theta}\mathbb{E}_t\left[Z_{t+1}v'\left(D_{t+1}\right)\right]F_{kk}(k_{t+1},1), \tag{15}$$

where

$$\begin{aligned}&\mathbb{E}_t\left[Z_{t+1}v'\left(D_{t+1}\right)\right]\\ =&\int_0^{z_{t+1}^{idf}}\lambda\theta Z_{t+1}\left(\bar{R}-R_{t+1}\right)^{\theta-1}\,\mathrm{d}\mathcal{F}_{Z_{t+1}|Z_t}(z)+\int_{z_{t+1}^{idf}}^{\infty}\theta Z_{t+1}\left(R_{t+1}-\bar{R}\right)^{\theta-1}\,\mathrm{d}\mathcal{F}_{Z_{t+1}|Z_t=z_t}(z).\end{aligned} \tag{16}$$

The above expression uncovers how the variation of capital in equilibrium affects the total utility with loss aversion. First, if the household does not obtain loss aversion utility from expected gains, $\eta_t \equiv 0$, the competitive equilibrium is optimal, which directly results from the first welfare theorem. Second, as long as the household is loss averse, a decrease in capital from the equilibrium level heightens the welfare since the second-order derivative of the production function, F, is negative. The negative extra term suggests that the marginal benefit of augmenting capital gets smaller compared to the competitive equilibrium. The social planner is able to improve welfare by diminishing capital from the equilibrium level, as long as the household is loss averse. The following proposition summarizes the statement.

Proposition 1. *The competitive equilibrium is inefficient as long as the household is loss averse; besides, the equilibrium capital stock is higher than the constrained optimal level.*

As the planner reduces the capital stock, the expected return to capital increases. The expected equity premium increases, resulting in a higher gain-loss utility conditional on exogenous possibility of productivity. Consumption decreases as a consequence of lower output. Meanwhile, however, the household is willing to consume more given output due to loss aversion. Thus the household experiences a smaller welfare loss from lower consumption relative to the welfare gain from the higher gain-loss utility, which is shown in the numerical analysis.

From another perspective, the household chooses an inefficient allocation because of pecuniary externalities. The atomistic household considers factor prices as exogenous and allocates capital regardless of the effect of her action on the whole economy through the channel of prices. On the contrary, the social planner internalizes the price effect and moderates asset holdings to affect asset prices and improve the gain-loss utility.

The finding on inefficiency relies on the assumptions of a production economy and the gain-loss utility containing prices. If an endowment economy is assumed as in the work of Barberis et al. (2001), the household chooses an efficient allocation, unlike in a production economy. The first-order condition for the social planner's problem no longer has the extra term because of no production function. If I model the gain-loss utility with only allocations like consumption, inefficiency fails to exist since the household's problem and the social planner's problem share the same solution. The characterization for both problems appear similar to the one considering habit formation, with asymmetry as a major difference.

4 A Special Functional Form

This section discusses a special case of the S-shaped gain-loss utility. A few propositions based on the case show that loss aversion reduces investment.

4.1 A Linear Function with a Kink

When $\theta = 1$, a S-shaped gain-loss utility function turns into a linear function with a kink,

$$v(X_{t+1}) = \begin{cases} X_{t+1}, & \text{if } X_{t+1} \geq 0; \\ \lambda X_{t+1}, & \text{if } X_{t+1} < 0. \end{cases} \tag{17}$$

Given the stochastic process of productivity, I simplify the expression of $\mathbb{E}_t\left[v(D_{t+1})\right]$ in the fashion of certainty equivalence. Appendix A.1 describes the detail of the simplification as,

$$\begin{aligned} \mathbb{E}_t\left[v(D_{t+1})\right] =& F_k(k_{t+1},1) z_t^{\rho} e^{\frac{\sigma_z^2}{2}} \left[1 + (\lambda-1)\Phi\left(\frac{\ln z_{t+1}^{idf} - (\rho \ln z_t + \sigma_z^2)}{\sigma_z}\right)\right] + \\ &+ (1-\delta-\bar{R})\left[1 + (\lambda-1)\Phi\left(\frac{\ln z_{t+1}^{idf} - \rho \ln z_t}{\sigma_z}\right)\right], \end{aligned} \tag{18}$$

where $\Phi(\cdot)$ represents the cumulative distribution function of the standard normal distribution. Technically speaking, the computation of the conditional expectation avoids the discussion of the kink at the reference point so that it facilitates further analysis.

4.2 Comparative Statics of Loss Aversion Components

I study the comparative statics of loss aversion components on equilibrium allocations, especially, on capital, by differentiating the Euler equation (13) with respect to λ and η_t, respectively. The following proposition summarizes the result. I leave the proof in Appendix B.

Proposition 2. *Fixing the current state, (k_t, z_t), and other parameters,*

(1) as the loss aversion degree, λ, increases, the household reduces investment in risky assets;

(2) as the relative weight on the gain-loss utility, η_t, increases, the household reduces investment in risky assets if she expects a loss and vice versa.

The first part suggests that given the fact that a loss affects the household more than a gain, the fear of a harmful loss drives her to accumulate less risky assets in equilibrium. The second part implies a reduction in investment with a higher weight on the loss aversion utility if the household expects a loss conditional on t's productivity. A higher weight means that the household pays more attention to the gain-loss utility, so that an expected loss harms her welfare more.

The effect of the loss aversion degree, λ, on the entire dynamic process of endogenous state variables, $\{k_{t+1}\}_{t=0}^{\infty}$, can be further manifested if I set the path of aggregate productivity, $\{Z_t\}_{t=0}^{\infty}$, and the initial condition of capital stock, k_0. I obtain the following proposition demonstrated by Appendix C:

Proposition 3. *Fixing a path of aggregate productivity, $\{Z_t\}_{t=0}^{\infty}$, the initial condition of capital stock, k_0, and other parameters, an increase in the loss aversion degree, λ, reduces investment in risky assets in all periods.*

Suppose that two households live in the same economy, thus they face the same path of aggregate productivity. If they differ in loss aversion degrees, the above proposition states that the household with a higher loss aversion degree always invests less in risky assets.

5 Numerical Analysis

This section presents the numerical results on a comparison between the competitive equilibrium and the constrained optimum. I start from describing how I calibrate the model. I present the asymmetry over the business cycles to show why loss aversion should be considered seriously. I compare the competitive equilibrium and the constrained optimum and find the difference between them. The welfare loss from the constrained optimum to the equilibrium is relatively large with baseline calibration.

5.1 Calibration

I assume the consumption preference as a standard CRRA function, $u(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma}$, where γ determines the degree of risk aversion and $\gamma > 0$. The production function is a Cobb-Douglas production function, $Y_t = Z_t k_t^{\alpha} n_t^{1-\alpha}$, where α denotes the capital share of output and $\alpha \in (0, 1)$. As Barberis et al. (2001), I assume that $\eta_t = \eta \bar{c}_t^{-\gamma}$, where I interpret $\bar{c}_t$ as aggregate consumption taken as exogenous by the representative household. In equilibrium, $\bar{c}_t = c_t, \forall t$. I apply the linear function with a kink as shown in the last section. Though a S-shaped gain-loss function with $\theta > 1$ improves the quantitative behavior with prospects of only gains or only losses, loss aversion remains as long as the link exists. The linear form allows me to compute the equilibrium and the constrained optimum in a simple way without loss of generality.

Most of the parameter values that I use are in line with the yearly data of the United States or other common values in the literature. I set the capital share of income $\alpha = 0.36$. My selection of δ is 0.1, which is in accordance with the annual depreciation rate. The risk aversion degree, γ, is set to be 5.

Furthermore, the discount factor, β, is calibrated to be 0.97 so that in a non-loss-aversion economy, given the above parameters, the ratio of capital over output is roughly 2.7 in the deterministic steady state. I keep the discount factor unchanged in this section when I introduce loss aversion in the model since loss aversion plays no role in the deterministic steady state.

I assume that the autoregressive parameter for the technology shock, ρ_z, and the standard deviation for innovations, σ_z, are 0.81 and 0.04, respectively, in keeping with the real business cycle literature.

I set the loss aversion degree, λ, equal to 2.25, measured by Tversky and Kahneman (1992). I follow the same logic of Barberis and Huang (2001) to pin down the relative weight parameter, η. They claim that the disutility of consuming a dollar less equals the psychological disutility of losing one dollar in investment in equilibrium[1]. The value of η turns out to be 0.45. These two

[1] It implies that $-\beta\eta\bar{c}_t^{-\gamma}\lambda = -c_t^{-\gamma}, \forall t$. Since in equilibrium, $\bar{c}_t = c_t, \forall t$, $\eta = \dfrac{1}{\beta\lambda}$.

values are baseline calibration of the loss aversion parameters. Given above parameter values, I search for the value of $\bar{R}$, letting the mean of R_t equal to $\bar{R}$ in equilibrium. Table 1 summarizes the parameter values for the model.

Table 1: Parameter Values for Baseline Model

Parameters	**Values**	**Descriptions**
α	0.36	capital share of output
δ	0.1	depreciation rate
γ	5	risk aversion degree
β	0.97	discounted rate
ρ_z	0.81	autocorrelation of productivity
σ_z	0.04	standard deviation of productivity shock
λ	2.25	baseline loss aversion degree
η	0.45	baseline relative weight parameter
$\bar{R}$	1.03	mean of gross return to capital

5.2 Asymmetric Impacts of Shocks

This subsection picks consumption as a representative variable. Figure 1 exhibits the impulse responses of consumption after a one-standard-deviation positive shock and after a one-standard-deviation negative shock. I take the absolute values when I plot the curve with the negative shock to facilitate the comparison. Consumption reacts less to the negative shock, showing asymmetric effects of expansions and recessions, which habit formation is unable to produce. Chen (2015) generates the same pattern though she sets a fixed value of consumption as the reference point. Our studies show together that loss aversion provides a competitive way to account for the asymmetry over the business cycles.

5.3 Comparison of Equilibrium and Constrained Optimum

Table 2 reports the comparison of selected prices and allocations in long-run means. It also shows the welfare loss from the constrained optimum to the equilibrium measured by consumption equivalent variation. I construct the consumption equivalent, ce, such that in every period,

$$\frac{ce_t^{1-\gamma}}{1-\gamma} = \frac{c_t^{1-\gamma}}{1-\gamma} + \eta_t \beta \mathbb{E}_t \left[v(D_{t+1}) \right]. \tag{19}$$

Thus, the long-run mean of the consumption equivalent reflects the total welfare. I set the base as the long-run mean of the consumption equivalent at the optimum.

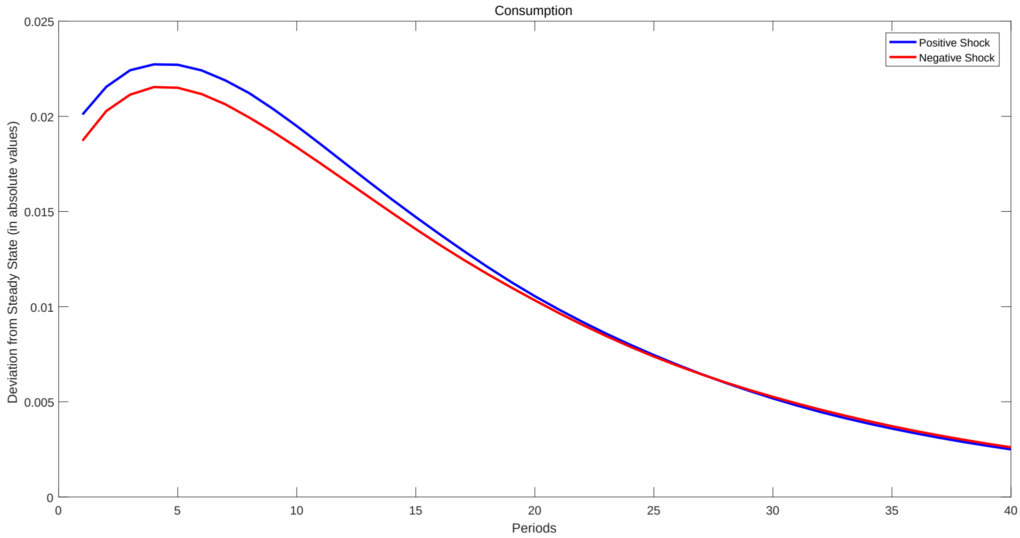

Figure 1: Impulse Responses of Consumption

Table 2: Comparison of Equilibrium and Optimum with Loss Aversion

	Baseline Calibration	
Variables	**Equilibrium**	**Optimum**
Capital	4.76	3.49
Output	1.76	1.57
Consumption	1.28	1.22
Capital Return	1.03	1.06
Consumption Share	73.17%	78.08%
Welfare Change (Consumption Equivalent Variation)	-0.33%	

Values of prices and allocations are in levels. The base of welfare change is set to be the long-run mean of the optimum.

In equilibrium, loss aversion magnifies the fluctuations in the investment in risky assets and in their returns. As analyzed before, capital stock in equilibrium with loss aversion, although lower than its counterpart in a standard model, is still higher than the constrained optimum. The household experiences a loss from lower consumption at the optimum, yet the reduction is relatively small since the fear of investment induces the household to consume more given output shown by “Consumption Share”. She obtains sufficient compensation from a higher gain-loss utility because the gross capital return rises 3%, driving the long-run mean of gain-loss utility from a negative value to a positive one . Therefore, she is better off by 0.33% measured by consumption equivalent variation when moving from the competitive equilibrium to the constrained optimum.

Without any risks, the social optimum coincides with the competitive equilibrium even if the household's preferences incorporate gains from investment because the modelling way of the gain-loss utility is based on uncertainty. Thus the numerical result from the baseline model also manifests that aggregate productivity shocks, or fluctuations over the business cycle, incur much more welfare loss than standard models generate. Lucas (1987) finds a negligible welfare gain if removing all the risks. Instead, the welfare gain is definitely nontrivial in my model because together with smooth consumption, the household no longer obtains disutility from potential losses from investment.

6 Policy Interventions

Inefficiency of competitive equilibrium due to loss aversion poses the questions of whether the government can intervene to reach the social optimum. This section answers the question by adding a government sector to the baseline model and exploring the implementation of allocations in the constrained optimum. I describe the government's problem directly with functional forms used in Section 5 to uncover the corresponding tax policy by quantitative analysis. I find that the government should tax capital to implement constrained optimal allocations.

6.1 Government

The government levies distortionary taxes on capital income at rate τ_t^k and rebates all the tax revenues in the lump-sum fashion. I assume that the tax rate, τ_t^k, is predetermined according to the information updated until period $t-1$, which implies that tax policy is non-state-contingent. This assumption reflects the norm of fiscal policy: policymakers usually propose and decide on a taxation policy before the policy enters into force. The non-state-contingent tax rate and state-contingent lump-sum subsidy make it possible to support a unique equilibrium. The government budget constraint is

$$\tau_t^k r_t k_t = T_t. \tag{20}$$

In a standard model, since the competitive equilibrium is optimal, the government naturally taxes no capital. I will show that the government in my model, on the contrary, has incentive to distort asset prices through capital income taxes to correct the inefficiency.

6.2 Household

At period t, the household receives income from labor supply, capital rental, interest on private bonds and government subsidy, is informed of the news of next period's taxation proposal and then determines the amount of consumption, labor supply and capital accumulation. The representative household compares the expected gross return and its average level in the fashion described in previous sections except that the household considers the gross return to capital net of capital income taxes, $R_t^k = (1 - \tau_t^k) r_t + 1 - \delta$. The indifferent productivity level to invest in risky assets and to receive income at a fixed interest rate is

$$z_{t+1}^{idf} = \frac{\bar{R} - 1 + \delta}{(1 - \tau_{t+1}^k) \alpha k_{t+1}^{\alpha - 1}}. \tag{21}$$

Providing that the government imposes a high tax on capital income, the value of z_{t+1}^{idf} will be large. Hence, a higher capital tax rate not only undermines the desire to accumulate capital, but also directly raises the expectation of losses.

With the government, the household budget constraint becomes

$$c_t + k_{t+1} = w_t n_t + \left[(1 - \tau_t^k) r_t + 1 - \delta \right] k_t + T_t. \tag{22}$$

6.3 Competitive Equilibrium Conditions

The determination of factor prices remains unchanged as in (11) and (12). The Euler equations incorporate identical terms as in (13) with the exception that the expected gain-loss utility per unit of capital, $\mathbb{E}_t \left[v(D_{t+1}) \right]$, given the tax rate, is rewritten as,

$$\begin{aligned} \mathbb{E}_t \left[v(D_{t+1}) \right] = \left(1 - \delta - \bar{R}\right) \left[1 + (\lambda - 1) \Phi \left(\frac{\ln z_{t+1}^{idf} - \rho \ln z_t}{\sigma_z} \right) \right] + \\ + (1 - \tau_{t+1}^k) \alpha k_{t+1}^{\alpha - 1} z_t^{\rho} e^{\frac{\sigma_z^2}{2}} \left[1 + (\lambda - 1) \Phi \left(\frac{\ln z_{t+1}^{idf} - (\rho \ln z_t + \sigma_z^2)}{\sigma_z} \right) \right]. \end{aligned} \tag{23}$$

6.4 Implementation of Constrained Optimal Allocations

This subsection reports how the government uses capital income taxes to implement the constrained optimal allocations.

6.4.1 Problem Formulation

The government aims to guide the market to implement constrained optimal allocations every period. Thus, the government decides on a sequence of capital income tax rates $\{\tau_{t+1}^k\}_{t=0}^{\infty}$ such that each time, the household chooses constrained optimal allocations as the solution to her maximization problem given tax policies, or equivalent, so that constrained optimal allocations let the Euler equation with respect to capital hold,

$$u'(c_t^O) = \beta \mathbb{E}_t\{(1-\tau_{t+1}^k)r_{t+1}^O + 1 - \delta]u'(c_{t+1}^O)\} + \eta_t \beta \mathbb{E}_t \left[v(D_{t+1}^O)\right], \tag{24}$$

where

$$D_{t+1}^O = (1-\tau_{t+1}^k)r_{t+1}^O + 1 - \delta - \bar{R}, \tag{25}$$

and

$$r_{t+1}^O = \alpha Z_t \left(k_t^O\right)^{\alpha-1}. \tag{26}$$

I index constrained optimal allocations and prices with the superscript of "O" in the above equations. I back out every period's capital income tax rate after inserting the values of constrained optimal allocations and prices calculated from the constrained social planner's problem addressed in previous sections.

6.4.2 Results on Policy Interventions

Table 3 reports the business cycle statistics of the capital income tax rate that implements the constrained optimal allocations. The mean is measured in percentage points. The relative standard deviation is computed by dividing its standard deviation in levels by its mean. As a comparison, Table 3 also records statistics of the same variables in a standard model.

Table 3: Statistics of Policy Instruments

Variables	**Baseline**		**Non-Loss-Aversion**	
	mean	**rsd.**	**mean**	**rsd.**
Capital Tax Rate	18.68	1.60	0	0

rsd. represents the relative standard deviation. The mean is measured in percentage points.

Table 3 presents my major finding on policy interventions: in an environment with loss aversion, the government should tax capital income. A zero capital tax rate is suboptimal. To overcome the inefficiency from loss aversion, the government reduces capital stock to implement constrained optimal allocations, thus it should tax capital considerably even without any idiosyncratic risks or frictions.

As for the relative standard deviations, the capital income tax rate in the standard model remains constant, equal to 0, over the whole business cycle. In the baseline model, the government applies it as an effective instrument to protect against the negative impact of fluctuations so that it varies a lot.

The result matches the argument of constrained efficient allocations. The government should apply its instruments, which are capital income taxes and lump-sum subsidies in my model, to reduce capital from an inefficient equilibrium level to a lower, optimal level. The planner faces a tradeoff between a lower level of consumption and a higher gain-loss utility. The latter dominates the former in this model.

6.4.3 Unable to Implement the Constrained Optimum

Readers may feel confused with the above claim, yet it does not contradict the previous statement that the government can implement constrained optimal allocations. The government can use capital taxation to lead the household to consume and augment capital stock as the optimum requires; it, however, fails to duplicate the same utility in the optimum because the gain-loss utility per unit of capital, $\mathbb{E}_t\left[v(D_{t+1})\right]$, with capital income taxes, is different from the utility without any policy intervention. Since the capital income tax rate that implements constrained optimal allocations is generally positive, the household receives lower utility than in the constrained optimum even if she allocates the same. Moreover, I find that the government is unable to implement the constrained optimum with any combination of capital income taxes and lump-sum subsidies as modelled in the above way.

7 The Model with Two Assets

This subsection augments the above model by adding another investment option – riskfree bonds. This section describes the household's problem and presents the solution. Appendix D shows that the constrained optimum is, in general, different from the market equilibrium, though a formal proof is unavailable in this scenario. I provide the comparison of the market equilibrium and the constrained optimum with numerical analysis. The equilibrium remains inefficient in this case.

7.1 Household

In the augmented model, a household can purchase non-state-contingent private bonds traded among individuals, in addition to consumption and capital accumulation. As a result, she receives income from bonds besides labor and capital income. The household maximizes her lifetime utility, (1), subject to budget constraints and nonnegativity constraints:

$$c_t + k_{t+1} + a_{t+1} = w_t n_t + r_t k_t + (1-\delta)k_t + R_t^f a_t, \tag{27}$$

$$c_t \geq 0,\ k_t \geq 0,\ 0 \leq n_t \leq 1. \tag{28}$$

R_t^f is the gross return to private bonds from $t-1$ to t, depending only on the state at $t-1$, so that individual assets are riskfree. When the household makes investment decisions, she undertakes risks if she augments capital stock while avoids risks if purchasing bonds.

X_{t+1} is assumed to have the form:

$$X_{t+1} = k_{t+1}R_{t+1}^k - k_{t+1}R_{t+1}^f. \tag{29}$$

At $t+1$, the household receives $k_{t+1}R_{t+1}^k$ from investment in risky assets. In fact, $k_{t+1}R_{t+1}^k$ is also the total financial wealth in equilibrium since the private bonds have zero net supply. I use the gross return to private bonds as the reference point for the household. $k_{t+1}R_{t+1}^f$ is the opportunity cost of investment in risky assets. If $R_{t+1}^k > R_{t+1}^f$, it is defined as a gain; if $R_{t+1}^k < R_{t+1}^f$, a loss. The equity premium is denoted as $D_{t+1} = R_{t+1}^k - R_{t+1}^f$, so I can interpret X_{t+1} as a product of the equity premium .

7.2 Solution to the Household's Problem

The Euler equations below characterize the solution to the household's maximization problem:

$$u'(c_t) = \beta \mathbb{E}_t \left[R_{t+1}^k u'(c_{t+1}) \right] + \eta_t \beta \mathbb{E}_t \left[v(D_{t+1}) \right], \tag{30}$$

$$u'(c_t) = \beta R_{t+1}^f \mathbb{E}_t \left[u'(c_{t+1}) \right]. \tag{31}$$

The indifferent cutoff of investing in two assets becomes: $z^{idf}_{t+1} = \frac{R^f_{t+1}-1+\delta}{F_k(k_{t+1},1)}$. z^{idf}_{t+1} will be larger if the agent accumulates more capital as previously. It is confirmed by differentiating z^{idf}_{t+1} with respect to capital, k_{t+1},

$$\frac{\mathrm{d}z^{idf}_{t+1}}{\mathrm{d}k_{t+1}} = \frac{F_k(k_{t+1},1)\frac{\mathrm{d}R^f_{t+1}}{\mathrm{d}k_{t+1}} - \left(R^f_{t+1} - 1 + \delta\right) F_{kk}(k_{t+1},1)}{\left(F_k(k_{t+1},1)\right)^2} > 0,$$

where

$$\frac{\mathrm{d}R^f_{t+1}}{\mathrm{d}k_{t+1}} = \frac{-u''(c_t) - \beta R^f_{t+1}\mathbb{E}_t\left[u''(c_{t+1})R^k_{t+1}\right]}{\beta\mathbb{E}_t\left[u'(c_{t+1})\right]} > 0.$$

As in the previous sections, I express $\mathbb{E}_t\left[v\left(D_{t+1}\right)\right]$ to simplify the analysis as[2],

$$\begin{aligned}\mathbb{E}_t\left[v(D_{t+1})\right] =& F_k(k_{t+1},1)z_t^{\rho}e^{\frac{\sigma_z^2}{2}}\left[1+(\lambda-1)\Phi\left(\frac{\ln z^{idf}_{t+1} - (\rho\ln z_t + \sigma_z^2)}{\sigma_z}\right)\right] + \\ &+ (1-\delta-R^f_{t+1})\left[1+(\lambda-1)\Phi\left(\frac{\ln z^{idf}_{t+1} - \rho\ln z_t}{\sigma_z}\right)\right].\end{aligned} \tag{32}$$

7.3 Comparison of Equilibrium and Constrained Optimum

Appendix D indicates that the constrained optimum is generally different from the market equilibrium. However, the result of inefficient equilibrium needs to be demonstrated. Furthermore, whether to raise or reduce the capital stock should be determined if inefficiency holds. Thus this subsection applies numerical analysis to show that given reasonable calibration, equilibrium is inefficient and has more capital than the constrained optimum requires.

7.3.1 Calibration

I aim to uncover the quantitative effect of loss aversion when the model generates a realistic equity premium. Thus I assume that the depreciation rate is a stochastic variable in the quantitative model instead of a parameter to generate a high equity premium, i.e., $\delta_t = \delta + \epsilon_\delta$, where $\epsilon_\delta \sim \mathcal{N}\left(0, \sigma_\delta^2\right)$. δ denotes the average depreciation rate while σ_δ captures the volatility of capital depreciation. The standard deviation of the depreciation rate, σ_δ, is determined to obtain the relative standard deviation of total consumption in the U.S. in the model with loss aversion. I choose the U.S. data on total consumption expenditure from 1929 to 2015 and find that the standard deviation of consumption accounts for about 57% of that of GDP. With the target,

[2] Appendix A.2 describes the detail of the simplification.

$\sigma_\delta = 0.067$. I keep other parameter values unchanged.

Table 4 reports the comparison of selected prices and allocations in long-run means. It also shows the welfare loss from the constrained optimum to the equilibrium measured by consumption equivalent variation.

Table 4: Comparison of Equilibrium and Optimum with Two Assets

Variables	**Equilibrium**	**Optimum**
Capital	4.17	3.26
Output	1.67	1.53
Consumption	1.25	1.20
Capital Return	1.05	1.07
Bond Return	1.03	1.01
Equity Premium	2.21%	5.83%
Welfare Change (Consumption Equivalent Variation)		
	-13.61%	

Values of prices and allocations are in levels. The base of welfare change is set to be the long-run mean of the optimum.

Note that the equity premium generated from the U.S. data is about 4 percentage points, yet it shows the levered case. A realistic unlevered equity premium should be around half of the above number as claimed by Croce (2014). In my model, even with a relatively low risk aversion degree ($\gamma = 5$) the unlevered equity premium in equilibrium reaches 2.21%. Loss aversion magnifies the fluctuations in the investment in risky assets and in their returns. Thus the equity premium needs to be sufficiently high to match riskier assets.

The model with two assets greatly enlarges the welfare gain from removing fluctuations. It manifests that the result of a relatively large welfare gain never depends on a constant reference point. Since the targeting return also becomes unstable, an adverse shock will raise the bond return and change the equity premium. Thus fluctuations hurt the household even more.

8 Conclusion

This paper discusses the behavior of a production economy over the business cycle considering loss aversion, a core concept of prospect theory commonly accepted in behavioral economics. As far as I know, research has scarcely applied prospect theory to a stochastic general equilibrium framework with a production economy. My paper models loss aversion as another component in addition to consumption in the household's preferences in a tractable way. The household acquires positive utility if she expects an extra gain from investing in risky assets, capital, relative to a reference point. If she predicts a loss, she gets disutility whose absolute value is greater than that of utility from the same amount of gain. Thus, fluctuations over the business cycle affects the welfare not only indirectly by making consumption volatile, but also directly by altering expectations on asset returns. The latter is generally negative due to the asymmetric influences of gains and losses on utility.

I show analytically that the competitive equilibrium is inefficient by considering a constrained optimality problem in the baseline model. The numerical analysis confirms the capability of my model to generate asymmetry over the business cycles, compares the competitive equilibrium and the constrained optimum and confirms the above statement in a quantitative model. This is because an atomic household takes prices as given and fails to think about the impact of her actions on prices. The pecuniary externality creates an inefficient equilibrium since prices enter into the preferences of a loss averse household. The optimal level of capital is much lower than the equilibrium level because the household can obtain a higher gain-loss utility from a higher equity premium if investing less in capital.

With a linear gain-loss utility function with a kink, I first focus on the comparative statics of loss aversion components and show that the more loss aversion and the more concern over the gain-loss utility, the less investment in risky assets in equilibrium given the current state and other parameters. Fixing a path of aggregate productivity, the initial condition of capital stock, and other parameters, an increase in the loss aversion degree reduces investment in risky assets in all periods.

I add the government sector and investigate the policy to implement the constrained optimal allocations. Zero capital tax in the long run becomes suboptimal. The government should tax capital accumulation to lower the capital stock.

I also find that the welfare loss from fluctuations over the business cycle is much higher than most of the estimations in standard models. This is because the household directly obtains disutility from potential losses from investment in addition to more volatile consumption.

Finally, a model with two assets functions as a robustness check, proving that the inefficiency of market equilibrium does not rely on the assumption of a constant reference point.

A Simplification of $\mathbb{E}_t\left[v(D_{t+1})\right]$

A.1 The case of a constant reference point

Since $\ln Z_{t+1} = \rho \ln Z_t + \sigma_z \epsilon^z_{t+1}$ and ϵ^z_{t+1} is distributed as a standard normal, Z_{t+1} follows a log-normal distribution conditional on t's information.We replicate here the expression of $\mathbb{E}_t\left[v(D_{t+1})\right]$,

$$\mathbb{E}_t v(D_{t+1}) = \int_0^{z^{idf}_{t+1}} \lambda(R_{t+1} - \bar{R})\,\mathrm{d}\mathcal{F}_{Z_{t+1}|Z_t=z_t}(z_{t+1}) + \int_{z^{idf}_{t+1}}^{\infty} (R_{t+1} - \bar{R})\,\mathrm{d}\mathcal{F}_{Z_{t+1}|Z_t=z_t}(z_{t+1}),$$

where the conditional cumulative distribution function of shock Z_{t+1} with the known history until period t, $\mathcal{F}_{Z_{t+1}|Z_t=z_t}(z_{t+1}) = \Phi\left(\frac{\ln z_{t+1} - \rho \ln z_t}{\sigma_z}\right)$ when $\Phi(\cdot)$ denotes the cumulative distribution function of the standard normal distribution. We can also derive the conditional probability density function as $f_{Z_{t+1}|Z_t=z_t}(z_{t+1}) = \frac{1}{\sigma_z z_{t+1}}\varphi\left(\frac{\ln z_{t+1} - \rho \ln z_t}{\sigma_z}\right)$ with $\varphi(\cdot)$ representing the probability density function of the standard normal distribution.

Let us focus on the first term of $\mathbb{E}_t\left[v(D_{t+1})\right]$.

$$\begin{aligned}
&\int_0^{z^{idf}_{t+1}} \lambda(R_{t+1} - \bar{R})\,\mathrm{d}\mathcal{F}_{Z_{t+1}|Z_t=z_t}(z_{t+1}) \\
&= \lambda \int_0^{z^{idf}_{t+1}} (z_{t+1} F_k(k_{t+1}, 1) + 1 - \delta - \bar{R})\,\mathrm{d}\mathcal{F}_{Z_{t+1}|Z_t=z_t}(z_{t+1}) \\
&= \lambda[\int_0^{z^{idf}_{t+1}} (1 - \delta - \bar{R})\,\mathrm{d}\mathcal{F}_{Z_{t+1}|Z_t=z_t}(z_{t+1}) + \int_0^{z^{idf}_{t+1}} F_k(k_{t+1}, 1)\cdot z_{t+1}\,\mathrm{d}\mathcal{F}_{Z_{t+1}|Z_t=z_t}(z_{t+1})] \\
&= \lambda[(1 - \delta - \bar{R})\Phi\left(\frac{\ln z^{idf}_{t+1} - \rho \ln z_t}{\sigma_z}\right) + F_k(k_{t+1}, 1)\int_0^{z^{idf}_{t+1}} \frac{z_{t+1}}{\sigma_z z_{t+1}}\varphi\left(\frac{\ln z_{t+1} - \rho \ln z_t}{\sigma_z}\right)\,\mathrm{d}z_{t+1}].
\end{aligned}$$

$$
\begin{aligned}
&\int_0^{z_{t+1}^{idf}} \frac{z_{t+1}}{\sigma_z z_{t+1}} \varphi\left(\frac{\ln z_{t+1}-\rho\ln z_t}{\sigma_z}\right)\,\mathrm{d}z_{t+1} \\
&= \int_0^{z_{t+1}^{idf}} \frac{1}{\sqrt{2\pi}\sigma_z} e^{-\frac{(\ln z_{t+1}-\rho\ln z_t)^2}{2\sigma_z^2}}\,\mathrm{d}z_{t+1} \\
&= \int_{-\infty}^{\ln z_{t+1}^{idf}} \frac{1}{\sqrt{2\pi}\sigma_z} e^{-\frac{(y_{t+1}-\rho\ln z_t)^2}{2\sigma_z^2}} e^{y_{t+1}}\,\mathrm{d}y_{t+1} (\text{Let } \ln z_{t+1} = y_{t+1}) \\
&= \int_{-\infty}^{\ln z_{t+1}^{idf}} \frac{1}{\sqrt{2\pi}\sigma_z} e^{-\frac{(y_{t+1}-(\rho\ln z_t+\sigma_z^2))^2}{2\sigma_z^2}+\rho\ln z_t+\frac{\sigma_z^2}{2}}\,\mathrm{d}y_{t+1} \\
&= z_t^{\rho} e^{\frac{\sigma_z^2}{2}} \int_{-\infty}^{\ln z_{t+1}^{idf}} \frac{1}{\sqrt{2\pi}\sigma_z} e^{-\frac{(y_{t+1}-(\rho\ln z_t+\sigma_z^2))^2}{2\sigma_z^2}}\,\mathrm{d}y_{t+1} \\
&= z_t^{\rho} e^{\frac{\sigma_z^2}{2}} \Phi\left(\frac{\ln z_{t+1}^{idf}-(\rho\ln z_t+\sigma_z^2)}{\sigma_z}\right).
\end{aligned}
$$

Thus,

$$
\begin{aligned}
&\int_0^{z_{t+1}^{idf}} \lambda(R_{t+1}-\bar{R})\,\mathrm{d}\mathcal{F}_{Z_{t+1}|Z_t=z_t}(z_{t+1}) \\
&= \lambda\left[(1-\delta-\bar{R})\Phi\left(\frac{\ln z_{t+1}^{idf}-\rho\ln z_t}{\sigma_z}\right) + F_k(k_{t+1},1) z_t^{\rho} e^{\frac{\sigma_z^2}{2}} \Phi\left(\frac{\ln z_{t+1}^{idf}-(\rho\ln z_t+\sigma_z^2)}{\sigma_z}\right)\right].
\end{aligned}
$$

With the same argument, we calculate the second term as well. Summing up two parts gives us the result:

$$
\begin{aligned}
\mathbb{E}_t\left[v(D_{t+1})\right] =& F_k(k_{t+1},1) z_t^{\rho} e^{\frac{\sigma_z^2}{2}} \left[1+(\lambda-1)\Phi\left(\frac{\ln z_{t+1}^{idf}-(\rho\ln z_t+\sigma_z^2)}{\sigma_z}\right)\right] + \\
&+ (1-\delta-\bar{R})\left[1+(\lambda-1)\Phi\left(\frac{\ln z_{t+1}^{idf}-\rho\ln z_t}{\sigma_z}\right)\right].
\end{aligned}
$$

A.2 The case of a time-varying reference point

The argument is similar to the previous argument with the only difference in the reference point. I report several key results.

$$
\begin{aligned}
&\int_0^{z_{t+1}^{idf}} \lambda(R_{t+1}^k - R_{t+1}^f)\,\mathrm{d}\mathcal{F}_{Z_{t+1}|Z_t=z_t}(z_{t+1}) \\
&= \lambda \int_0^{z_{t+1}^{idf}} (z_{t+1}F_k(k_{t+1},1) + 1 - \delta - R_{t+1}^f)\,\mathrm{d}\mathcal{F}_{Z_{t+1}|Z_t=z_t}(z_{t+1}) \\
&= \lambda[\int_0^{z_{t+1}^{idf}} (1 - \delta - R_{t+1}^f)\,\mathrm{d}\mathcal{F}_{Z_{t+1}|Z_t=z_t}(z_{t+1}) + \int_0^{z_{t+1}^{idf}} F_k(k_{t+1},1)\cdot z_{t+1}\,\mathrm{d}\mathcal{F}_{Z_{t+1}|Z_t=z_t}(z_{t+1})] \\
&= \lambda[(1-\delta-R_{t+1}^f)\Phi\left(\frac{\ln z_{t+1}^{idf} - \rho\ln z_t}{\sigma_z}\right) + F_k(k_{t+1},1)\int_0^{z_{t+1}^{idf}} \frac{z_{t+1}}{\sigma_z z_{t+1}}\varphi\left(\frac{\ln z_{t+1} - \rho\ln z_t}{\sigma_z}\right)\mathrm{d}z_{t+1}].
\end{aligned}
$$

$$
\begin{aligned}
&\int_0^{z_{t+1}^{idf}} \frac{z_{t+1}}{\sigma_z z_{t+1}}\varphi\left(\frac{\ln z_{t+1} - \rho\ln z_t}{\sigma_z}\right)\mathrm{d}z_{t+1} \\
&= \int_0^{z_{t+1}^{idf}} \frac{1}{\sqrt{2\pi}\sigma_z} e^{-\frac{(\ln z_{t+1} - \rho\ln z_t)^2}{2\sigma_z^2}}\,\mathrm{d}z_{t+1} \\
&= \int_{-\infty}^{\ln z_{t+1}^{idf}} \frac{1}{\sqrt{2\pi}\sigma_z} e^{-\frac{(y_{t+1} - \rho\ln z_t)^2}{2\sigma_z^2}} e^{y_{t+1}}\,\mathrm{d}y_{t+1} (\text{Let } \ln z_{t+1} = y_{t+1}) \\
&= \int_{-\infty}^{\ln z_{t+1}^{idf}} \frac{1}{\sqrt{2\pi}\sigma_z} e^{-\frac{(y_{t+1} - (\rho\ln z_t + \sigma_z^2))^2}{2\sigma_z^2} + \rho\ln z_t + \frac{\sigma_z^2}{2}}\,\mathrm{d}y_{t+1} \\
&= z_t^\rho e^{\frac{\sigma_z^2}{2}} \int_{-\infty}^{\ln z_{t+1}^{idf}} \frac{1}{\sqrt{2\pi}\sigma_z} e^{-\frac{(y_{t+1} - (\rho\ln z_t + \sigma_z^2))^2}{2\sigma_z^2}}\,\mathrm{d}y_{t+1} \\
&= z_t^\rho e^{\frac{\sigma_z^2}{2}} \Phi\left(\frac{\ln z_{t+1}^{idf} - (\rho\ln z_t + \sigma_z^2)}{\sigma_z}\right).
\end{aligned}
$$

$$
\begin{aligned}
&\int_0^{z_{t+1}^{idf}} \lambda(R_{t+1}^k - R_{t+1}^f)\,\mathrm{d}\mathcal{F}_{Z_{t+1}|Z_t=z_t}(z_{t+1}) \\
&= \lambda\left[(1-\delta-R_{t+1}^f)\Phi\left(\frac{\ln z_{t+1}^{idf} - \rho\ln z_t}{\sigma_z}\right) + F_k(k_{t+1},1) z_t^\rho e^{\frac{\sigma_z^2}{2}} \Phi\left(\frac{\ln z_{t+1}^{idf} - (\rho\ln z_t + \sigma_z^2)}{\sigma_z}\right)\right].
\end{aligned}
$$

The whole expectation of expected gains is:

$$
\begin{aligned}
\mathbb{E}_t\left[v(D_{t+1})\right] =& F_k(k_{t+1},1) z_t^\rho e^{\frac{\sigma_z^2}{2}} \left[1 + (\lambda-1)\Phi\left(\frac{\ln z_{t+1}^{idf} - (\rho\ln z_t + \sigma_z^2)}{\sigma_z}\right)\right] + \\
&+ (1-\delta-R_{t+1}^f)\left[1 + (\lambda-1)\Phi\left(\frac{\ln z_{t+1}^{idf} - \rho\ln z_t}{\sigma_z}\right)\right].
\end{aligned}
$$

B The Proof of Proposition 2

Proof. I differentiate the Euler equation (13) with respect to λ and η_t, respectively.

$$\begin{aligned}
&- u''(c_t)\frac{\mathrm{d}k_{t+1}}{\mathrm{d}\lambda} \\
&= \beta\mathbb{E}_t\left[u''(c_{t+1})R_{t+1}^2 + u'(c_{t+1})Z_{t+1}F_{kk}(k_{t+1},1)\right]\frac{\mathrm{d}k_{t+1}}{\mathrm{d}\lambda}+ \\
&+\eta\beta\left\{\int_0^{z_{t+1}^{idf}}\left[\left(R_{t+1}-\bar{R}\right)+\lambda Z_{t+1}F_{kk}(k_{t+1},1))\frac{\mathrm{d}k_{t+1}}{\mathrm{d}\lambda}\right]\mathrm{d}\mathcal{F}_{Z_{t+1}|Z_t=z_t}(z)+\right. \\
&+\int_{z_{t+1}^{idf}}^{\infty} Z_{t+1}F_{kk}(k_{t+1},1)\frac{\mathrm{d}k_{t+1}}{\mathrm{d}\lambda}\,\mathrm{d}\mathcal{F}_{Z_{t+1}|Z_t=z_t}(z)+ \\
&\left.+(\lambda-1)\left(z_{t+1}^{idf}F_k(k_{t+1},1)+1-\delta-\bar{R}\right)\frac{\mathrm{d}z_{t+1}^{idf}}{\mathrm{d}k_{t+1}}\frac{\mathrm{d}k_{t+1}}{\mathrm{d}\lambda}\right\}.
\end{aligned} \tag{33}$$

$\left(z_{t+1}^{idf}F_k(k_{t+1},1)+1-\delta-\bar{R}\right)$ equals 0 after evaluating the value of the cutoff z_{t+1}^{idf}, so I cross out the last line. From the above equation, I obtain

$$\begin{aligned}
&\frac{\mathrm{d}k_{t+1}}{\mathrm{d}\lambda} \\
&= \eta\beta\int_0^{z_{t+1}^{idf}}\left(R_{t+1}-\bar{R}\right)\mathrm{d}\mathcal{F}_{Z_{t+1}|Z_t=z_t}(z) \div \left\{-\beta\mathbb{E}_t\left[u''(c_{t+1})R_{t+1}^2+u'(c_{t+1})Z_{t+1}F_{kk}(k_{t+1},1)\right]-\right. \\
&\left.-\eta\beta\left\{F_{kk}(k_{t+1},1)z_t^{\rho}e^{\frac{\sigma_z^2}{2}}\left[1+(\lambda-1)\Phi\left(\frac{\ln z_{t+1}^{idf}-(\rho\ln z_t+\sigma_z^2)}{\sigma_z}\right)\right]\right\}-u''(c_t)\right\}.
\end{aligned} \tag{34}$$

The divisor is positive while the dividend is negative because expected productivity under the cutoff suggests a loss. The quotient is, as a result, negative.

Likewise,

$$\begin{aligned}
\frac{\mathrm{d}k_{t+1}}{\mathrm{d}\eta_t} &= \beta\mathbb{E}_t\left[v(D_{t+1})\right] \div \left\{-\beta\mathbb{E}_t\left[u''(c_{t+1})R_{t+1}^2+u'(c_{t+1})Z_{t+1}F_{kk}(k_{t+1},1)\right]-u''(c_t)-\right. \\
&\left.-\eta\beta\left\{F_{kk}(k_{t+1},1)z_t^{\rho}e^{\frac{\sigma_z^2}{2}}\left[1+(\lambda-1)\Phi\left(\frac{\ln z_{t+1}^{idf}-(\rho\ln z_t+\sigma_z^2)}{\sigma_z}\right)\right]\right\}\right\}.
\end{aligned} \tag{35}$$

The quotient has the same sign as the dividend. □

C The Proof of Proposition 3

Proof. Take arbitrarily two values of loss aversion degree, $1 < \lambda < \lambda'$. Since the initial state, (k_0, z_0), is fixed, according to Proposition 1, I compare capital stock in Period 1:

$$k_1'(k_0, z_0) < k_1(k_0, z_0).$$

Applying Proposition 1 again yields the comparison of capital stock in Period 2:

$$k_2'(k_1, z_1) < k_2(k_1, z_1).$$

I duplicate the Euler equation in Period 1:

$$\begin{aligned} u'(c_1) &= \beta \mathbb{E}_1 \left[R_2 u'(c_2) \right] + \eta \beta \mathbb{E}_1 \left[v(D_2) \right] \\ &= \beta \mathbb{E}_1 \left[R_2 u'(c_2) \right] + \eta \beta \mathbb{E}_1 \left[\int_0^{z_2^{idf}} \lambda \left(R_2 - \bar{R} \right) \mathrm{dF}_{Z_2 | Z_1 = z_1}(z) + \int_{z_2^{idf}}^{\infty} \left(R_2 - \bar{R} \right) \mathrm{dF}_{Z_2 | Z_1 = z_1}(z) \right]. \end{aligned}$$

Differentiating the Euler equation with respect to k_1 yields

$$\begin{aligned} u''(c_1) \left(R_1 - \frac{\mathrm{d}k_2}{\mathrm{d}k_1} \right) = {} & \beta \mathbb{E}_1 \left[R_2^2 u''(c_2) + u'(c_2) Z_2 F_{kk}(k_2, 1) \right] \frac{\mathrm{d}k_2}{\mathrm{d}k_1} + \\ & + \eta \beta \left[\int_0^{z_2^{idf}} \lambda Z_2 F_{kk}(k_2, 1)) \frac{\mathrm{d}k_2}{\mathrm{d}k_1} \mathrm{dF}_{Z_2 | Z_1 = z_1}(z) + \right. \\ & + \int_{z_2^{idf}}^{\infty} Z_2 F_{kk}(k_2, 1) \frac{\mathrm{d}k_2}{\mathrm{d}k_1} \mathrm{dF}_{Z_2 | Z_1 = z_1}(z) + \\ & \left. + (\lambda - 1) \left(z_2^{idf} F_k(k_2, 1) + 1 - \delta - \bar{R} \right) \frac{\mathrm{d}z_2^{idf}}{\mathrm{d}k_2} \frac{\mathrm{d}k_2}{\mathrm{d}k_1} \right]. \end{aligned}$$

I cross the last line because by definition, $z_2^{idf} = \frac{\bar{R} - 1 + \delta}{F_k(k_2, 1)}$. Then organizing the equation obtains

$$\begin{aligned} \frac{\mathrm{d}k_2}{\mathrm{d}k_1} = {} & u''(c_1) R_1 \div \left\{ u''(c_1) + \beta \mathbb{E}_1 \left[R_2^2 u''(c_2) + u'(c_2) Z_2 F_{kk}(k_2, 1) \right] + \right. \\ & \left. + \eta \beta \left[\int_0^{z_2^{idf}} \lambda Z_2 F_{kk}(k_2, 1)) \mathrm{dF}_{Z_2 | Z_1 = z_1}(z) + \int_{z_2^{idf}}^{\infty} Z_2 F_{kk}(k_2, 1) \mathrm{dF}_{Z_2 | Z_1 = z_1}(z) \right] \right\} \end{aligned}$$

Both the divisor and the dividend are negative because of strictly concave utility and production functions, thus the quotient is positive. It indicates that given parameters and the path of aggregate productivity, the capital stock in Period 2 increases with capital in Period 1.

Since $k'_1(k_0, z_0) < k_1(k_0, z_0)$, $k'_2(k'_1, z_1) < k'_2(k_1, z_1)$.

Therefore, $k'_2(k'_1, z_1) < k_2(k_1, z_1)$ by transitivity.

Iterating the process obtains

$$k'_{t+1}(k'_t, z_t) < k_{t+1}(k_t, z_t).$$

Hence, summarizing the whole dynamic path yields

$$k'_{t+1}(z_t; \lambda', k_0) < k_{t+1}(z_t; \lambda, k_0).$$

$\square$

D Discussion of Efficiency in the Augmented Model

This section discusses the efficiency of competitive equilibrium in the augmented model. I first construct a simplified two-period model to analyze the effect of increasing the capital stock on utility following Davila et al. (2012). It indicates that in the two-period model the competitive equilibrium is surely inefficient when the agent is loss averse. I then return to an infinite-period model and compare the allocations in the competitive equilibrium and the constrained optimum. The result shows that in general, the competitive equilibrium differs from the constrained optimum in its characterization.

D.1 Inefficient Equilibrium in a Two-Period Model

Consider an economy with a continuum of homogeneous households that live two periods. In the first period, period 0, a household is endowed with e units of output and chooses the amount of consumption, c_0, capital stock, k, and bonds, a, to maximize her utility from consumption and loss aversion utility from expected gains. In the second period, period 1, she obtains utility only from period 1's consumption after receiving labor and asset income. In period 0, aggregate productivity is normalized to 1; in period 1, a productivity shock, σ_z, hits the economy, causing stochastic productivity Z_1.

The representative firm uses labor and capital, and produces output with a constant-returns-to-scale Cobb-Douglas production technology. As analyzed previously, the household supplies her labor completely, $n = 1$, in equilibrium. The wage and the rental rate equal the marginal products of inputs with labor evaluated at 1. The household holds zero bond in equilibrium, $a = 0$.

The competitive equilibrium in this two-period model is a sequence of prices $\{w_1, r_1, R^f\}$ and a sequence of allocations $\{k, a\}$ such that

(1) given prices, k and a solve

$$\max_{k,a} u(e - k - a) + \eta_0 \beta \mathbb{E}_0 \left[v \left(R_1 - R^f \right) k \right] + \beta \mathbb{E}_0 \left[u \left(w_1 + R_1 k + R^f a \right) \right];$$

(2) $r_1 = z_1 F_k(k, 1)$ and $w_1 = z_1 F_n(k, 1)$;

(3) $a = 0$.

The social planner chooses an allocation, meaning that they can only adjust the level of capital stock to affect welfare. I denote the total utility across two periods as U. I define the constrained efficiency of an allocation as follows:

Definition. *The allocation k is constrained efficient if it is feasible (i.e., $k \in [0, e]$) and if there is no other feasible allocation k' such that $U(k') > U(k)$.*

Whether the competitive equilibrium is constrained efficient depends on whether the planner can improve welfare by dictating a different level of capital. Thus I consider the effect of increasing capital on the total utility following Davila et al. (2012). Differentiating the total utility, I obtain

$$\begin{aligned} \mathrm{d}U = & -u'(e-k-a)(\mathrm{d}k+\mathrm{d}a)+ \\ & + \eta_0\beta\left\{\mathbb{E}_0\left[v\left(R_1-R^f\right)\right]\mathrm{d}k + k\mathbb{E}_0\left[\frac{\mathrm{d}v\left[\left(R_1-R^f\right)\right]}{\mathrm{d}R_1} + \frac{\mathrm{d}\left[v\left(R_1-R^f\right)\right]}{\mathrm{d}R^f}\right]\right\} + \\ & + \beta\mathbb{E}_0\left[u'\left(w_1+R_1k+R^fa\right)\left(\mathrm{d}w_1+R_1\mathrm{d}k+k\mathrm{d}R_1+R^f\mathrm{d}a+a\mathrm{d}R^f\right)\right] \end{aligned}$$

The first-order conditions for the household's maximization problem read

$$u'(e-k-a) = \beta\mathbb{E}_0\left[u'(w_1+R_1k+R^fa)R_1\right] + \eta_0\beta\mathbb{E}_0\left[v\left(R_1-R^f\right)\right],$$

$$u'(e-k-a) = \beta\mathbb{E}_0\left[u'(w_1+R_1k+R^fa)\right]R^f.$$

I simplify the expression of $\mathrm{d}U$ by inserting these conditions and then obtain

$$\begin{aligned} \mathrm{d}U = & \eta_0\beta\left\{\mathbb{E}_0\left[v\left(R_1-R^f\right)\right]\mathrm{d}k + k\mathbb{E}_0\left[\frac{\mathrm{d}v\left[\left(R_1-R^f\right)\right]}{\mathrm{d}R_1} + \frac{\mathrm{d}\left[v\left(R_1-R^f\right)\right]}{\mathrm{d}R^f}\right]\right\} + \\ & + \beta\mathbb{E}_0\left[u'\left(w_1+R_1k+R^fa\right)\left(\mathrm{d}w_1+k\mathrm{d}R_1+a\mathrm{d}R^f\right)\right]. \end{aligned}$$

Note that $\mathrm{d}R_1 = \mathrm{d}r_1 = z_1F_{kk}(k,1)\mathrm{d}k$ and $\mathrm{d}w_1 = z_1F_{nk}(k,1)\mathrm{d}k$. Since the production technology, F, is homogeneous of degree 1, $F_n(k,1) + kF_k(k,1) = F(k,1)$. Differentiating both handsides with respect to k, we obtain $F_{nk}(k,1)\mathrm{d}k + kF_{kk}(k,1)\mathrm{d}k = 0$. Therefore,

$$\mathrm{d}w_1 + k\mathrm{d}R_1 = z_1F_{nk}(k,1)\mathrm{d}k + z_1kF_{kk}(k,1)\mathrm{d}k = 0.$$

I focus on the efficiency of equilibrium, and thus examine the impact of a small deviation from equilibrium. After inserting market equilibrium conditions and the above expressions, I differentiate the return to riskfree assets from the first-order condition with respect to capital evaluating $a = 0$, and get

$$\frac{\mathrm{d}R^f}{\mathrm{d}k} = \frac{-u''(e-k) - \beta R^f \mathbb{E}_0\left[u''(w_1 + R_1 k) R_1\right]}{\beta \mathbb{E}_0\left[u'(w_1 + R_1 k)\right]} > 0, \tag{36}$$

$$\begin{aligned}
&\frac{\mathrm{d}U\mid_{\text{equilibrium}}}{\mathrm{d}k} \\
&= \eta_0 \beta k \left[\int_0^{z^{idf}} \lambda \left(\frac{\mathrm{d}R^k}{\mathrm{d}k} - \frac{\mathrm{d}R^f}{\mathrm{d}k} \right) \mathrm{dF}_{Z_1|Z_0=1}(z) + \lambda \left(z^{idf} F_k(k,1) + 1 - \delta - R^f \right) \frac{\mathrm{d}z^{idf}}{\mathrm{d}k} + \right. \\
&\qquad \left. + \int_{z^{idf}}^{\infty} \left(\frac{\mathrm{d}R^k}{\mathrm{d}k} - \frac{\mathrm{d}R^f}{\mathrm{d}k} \right) \mathrm{dF}_{Z_1|Z_0=1}(z) - \left(z^{idf} F_k(k,1) + 1 - \delta - R^f \right) \frac{\mathrm{d}z^{idf}}{\mathrm{d}k} \right] \\
&= \eta_0 \beta k \left\{ \left[1 + (\lambda - 1) \Phi \left(\frac{\ln z^{idf} - \sigma_z^2}{\sigma_z} \right) \right] e^{\frac{\sigma_z^2}{2}} F_{kk}(k,1) - \left[1 + (\lambda - 1) \Phi \left(\frac{\ln z^{idf}}{\sigma_z} \right) \right] \frac{\mathrm{d}R^f}{\mathrm{d}k} \right\}.
\end{aligned} \tag{37}$$

$\left(z^{idf} F_k(k,1) + 1 - \delta - R^f\right)$ equals 0 after evaluating the expression of cutoff, z^{idf}, which implies that a change in capital does not affect the total utility through the channel of changing the cutoff value. These two terms in the last line are both negative. It manifests that a reduction in capital from the equilibrium level exerts a positive impact on the welfare in this two-period model.

The above expression uncovers how the variation of capital in equilibrium affects the total utility with loss aversion. First, if the household does not obtain loss aversion utility from expected gains, $\eta_0 = 0$, the competitive equilibrium is optimal, which directly results from the first welfare theorem. Second, as long as the household is loss averse, a decrease in capital from the equilibrium level heightens the welfare since the second-order derivatives of the production function, F, and the utility from consumption, u, are negative. The following proposition summarizes the statement.

Proposition 4. *In a two-period augmented model, the competitive equilibrium is suboptimal as long as the household is loss averse; besides, the equilibrium capital stock is higher than the constrained optimal level.*

D.2 Characterization of Constrained Efficiency

I provide the necessary condition of constrained efficiency, the first-order condition of the household's maximization problem subject to the feasibility constraint. Following the same notation in the last subsection, $\frac{\mathrm{d}U}{\mathrm{d}k} = 0$. I characterize the constrained efficiency for the augmented model with infinite periods. The constrained optimum is, in general, different from the competitive equilibrium.

The constrained efficient social planner maximizes the household's lifetime utility subject to the feasibility constraint and the pricing rule as in the competitive market.

$$\max_{c_t, k_{t+1}} \quad u(c_t) + \eta_t \beta k_{t+1} \mathbb{E}_t \left[v(D_{t+1}) \right]$$

subject to

$$c_t + k_{t+1} = Z_t F(k_t, 1) + (1 - \delta) k_t,$$

where

$$R^f_{t+1} = \frac{u'(c_t)}{\beta \mathbb{E}_t \left[u'(c_{t+1}) \right]} \text{ and } D_{t+1} = Z_{t+1} F_k(k_{t+1}, 1) + 1 - \delta - R^f_{t+1}.$$

The first-order condition for the social planner's problem is

$$\begin{aligned} u'(c_0) = {} & \beta \mathbb{E}_0 \left[R^k_1 u'(c_1) \right] + \eta_0 \beta \mathbb{E}_0 \left[v(D_1) \right] + \\ & + \eta_0 \beta k_1 \left\{ \left[1 + (\lambda - 1) \Phi \left(\frac{\ln z^{idf}_1 - (\rho \ln z_0 + \sigma^2_z)}{\sigma_z} \right) \right] z_0^{\rho} e^{\frac{\sigma^2_z}{2}} F_{kk}(k_1, 1) - \right. \\ & \left. - \left[1 + (\lambda - 1) \Phi \left(\frac{\ln z^{idf}_1 - \rho \ln z_0}{\sigma_z} \right) \right] \frac{\mathrm{d}R^f_1}{\mathrm{d}k_1} \right\} - \\ & - \beta^2 \mathbb{E}_0 \left\{ \eta_1 k_2 \left[1 + (\lambda - 1) \Phi \left(\frac{\ln z^{idf}_2 - \rho \ln z_1}{\sigma_z} \right) \right] \frac{\mathrm{d}R^f_2}{\mathrm{d}k_1} \right\}; \end{aligned} \tag{38}$$

$$\begin{aligned} u'(c_t) = {} & \beta \mathbb{E}_t \left[R^k_{t+1} u'(c_{t+1}) \right] + \eta_t \beta \mathbb{E}_t \left[v(D_{t+1}) \right] + \\ & + \eta_t \beta k_{t+1} \left\{ \left[1 + (\lambda - 1) \Phi \left(\frac{\ln z^{idf}_{t+1} - (\rho \ln z_t + \sigma^2_z)}{\sigma_z} \right) \right] z_t^{\rho} e^{\frac{\sigma^2_z}{2}} F_{kk}(k_{t+1}, 1) - \right. \\ & \left. - \left[1 + (\lambda - 1) \Phi \left(\frac{\ln z^{idf}_{t+1} - \rho \ln z_t}{\sigma_z} \right) \right] \frac{\mathrm{d}R^f_{t+1}}{\mathrm{d}k_{t+1}} \right\} - \\ & - \beta^2 \mathbb{E}_t \left\{ \eta_{t+1} k_{t+2} \left[1 + (\lambda - 1) \Phi \left(\frac{\ln z^{idf}_{t+2} - \rho \ln z_{t+1}}{\sigma_z} \right) \right] \frac{\mathrm{d}R^f_{t+2}}{\mathrm{d}k_{t+1}} \right\} - \\ & - \eta_{t-1} k_t \left[1 + (\lambda - 1) \Phi \left(\frac{\ln z^{idf}_t - \rho \ln z_{t-1}}{\sigma_z} \right) \right] \frac{\mathrm{d}R^f_t}{\mathrm{d}k_{t+1}}, \quad \forall t \geq 1 \end{aligned} \tag{39}$$

where

$$\frac{\mathrm{d}R^f_{t+1}}{\mathrm{d}k_{t+1}} = \frac{-u''(c_t) - \beta R^f_{t+1} \mathbb{E}_t \left[u''(c_{t+1}) R^k_{t+1} \right]}{\beta \mathbb{E}_t \left[u'(c_{t+1}) \right]} > 0, \quad \forall t \geq 0$$

$$\frac{\mathrm{d}R_{t+2}^f}{\mathrm{d}k_{t+1}} = \frac{u''(c_{t+1})\,R_{t+1}^k}{\beta\mathbb{E}_{t+1}\left[u'(c_{t+2})\right]} < 0, \quad \forall t \geq 0$$

$$\frac{\mathrm{d}R_t^f}{\mathrm{d}k_{t+1}} = \frac{u'(c_{t-1})\,u''(c_t)}{\beta\mathbb{E}_{t-1}^2\left[u'(c_t)\right]} < 0, \quad \forall t \geq 1.$$

The social planner's choice exhibits a time-inconsistent problem. What the planner should do in period 0 differs from what he should do in the following periods. The reason why time-inconsistency appears lies in the interest rate of riskfree bonds functioning as the reference point.

Note that the first line of (39) duplicates the equilibrium condition, yet there are more terms in the optimum characterization. Line 2 and 3 suggest the same effect of a deviation from a certain capital level as in the two-period model. Line 4 shows that a different level of capital, k_{t+1}, affects the gain-loss utility in period $t+1$ since capital then will evolve through a different path, resulting in a different capital level decided in $t+1$, k_{t+2}, leading to different returns to bonds. The last line indicates that a change in the allocation in the future even influences the gain-loss utility in the past in a perfect foresight model because the interest rate of bonds varies if modifying future consumption, which further affects past consumption. The planner, however, does not have to consider the last effect in period 0, thus time-inconsistency shows up.

Hence, increasing capital, k_{t+1}, lowers t's gain-loss utility while it raises the gain-loss utility in $t+1$ and in $t-1$ since returns to safe assets decline accordingly. The positive influence of higher capital exerted in $t+1$ and $t-1$ makes the optimization problem more complicated to analyze than in a two-period model. I conclude that the constrained optimum is generally different from the market equilibrium in the infinite-horizon model with two assets.

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