When to Tax Capital: Fiscal Policy with Idiosyncratic Investment Risks and Heterogeneous Agents*

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Abstract

This paper examines the effect of fiscal policies on the investment and the welfare of heterogeneous agents over the business cycle. I embed business cycles and a government into the framework of Angeletos (2007) with independent and identically distributed idiosyncratic investment risks whose volatility is assumed to be countercyclical. All entrepreneurs make identical saving and portfolio choices each period, allowing for exact aggregation which facilitates computation. The model matches income inequality and dynamics of the income distribution over the cycle compared to the US data. The government sets rules of capital income tax rate and debt as functions of current output fixing the steady state level. I calibrate the cyclicality of fiscal policy to the US data as the baseline and adjust the parameters indicating cyclicality to study the effect of counterfactual policy. A combination of capital tax and debt policy creates a welfare conflict between entrepreneurs and workers. The policy, which optimizes utilitarian social utility, specifies that the capital income tax rate should increase by 0.45% and the debt should decrease by 0.59% facing a 1% decrease in output. It is possible that the government should reduce the capital tax in the recession if it increases the weight of workers on social welfare. The impulse response of aggregate variables to a negative productivity shock indicates that in general, the higher capital tax rate and the less debt when the adverse shock hits, the more capital and output in the early stage after the shock. The result of the welfare conflict is robust to constant idiosyncratic investment risks, but not to the removal of idiosyncratic investment risks.

JEL classification: E25; E32; E62

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1 Introduction

Empirical studies find that wealth is highly concentrated among the rich. Heathcote, Perri and Violante (2010) document that the richest 10 percent of households in the US hold 59 percent of aggregate net worth in 2007. The net worth tends to be invested in an undiversified portfolio, even for the rich who in general have more investment options. Carroll (2000) find that portfolios of the rich are heavily skewed toward risky assets, especially investment in their own private-held businesses. Poor diversification in investment implies high idiosyncratic investment risks, which affect saving and investment choices, the aggregate economy and the welfare. Furthermore, the idiosyncratic investment risks increase in the recession, causing a larger difference in the performance of returns to investment as shown by Bloom et al (2014). It may amplify the damage to aggregate investment and output. Thus idiosyncratic investment risks may provide implications on adjusting the capital income tax or other policies related to investment over the business cycle.

This paper examines the effect of fiscal policies on the investment and on the welfare of heterogeneous agents over the business cycle with a model featuring the aforementioned facts about wealth and investment. Specifically, I ask when to increase the capital income tax rate. Due to idiosyncratic investment risks, an increase in the capital income tax rate may exert a larger impact on the investment and on the welfare of investors. The change in the tax policy further influences output, the wage rate and the welfare of agents who receive labor earnings as the main income source.

I develop a model that incorporates heterogeneous agents, business cycle and a government sector which decides fiscal policies in the framework of Angeletos (2007) considering idiosyncratic investment risks. My model features two types of risk-averse agents: entrepreneurs and hand-to-mouth workers. I allow entrepreneurs to invest in their own firms, or alternatively in non-state-contingent government bonds, but not in the private firms held by other entrepreneurs. Private firms suffer from aggregate productivity shocks and independent and identically distributed idiosyncratic investment risks whose variance rises in the recession, so that low total factor productivity enlarges the gap in business income. Together with i.i.d idiosyncratic investment risks, the aggregate shocks exert an impact on the decision of how much to consume and to save (saving choice) and of how much investment on risky but productive assets and on riskfree assets (portfolio choice). Each period all entrepreneurs make identical choices because of i.i.d idiosyncratic investment risks and no trade in capital among entrepreneurs. Hand-to-mouth workers supply labor inelastically to the labor market. Their income differs due to idiosyncratic labor income risks. A government levies labor and capital taxes and issues government bonds. Specifically, the government sets rules of capital tax and bonds as functions of log-deviation of output fixing the steady state level and lets the labor tax rate balance the budget. These policies influence entrepreneurs more directly since they can save.

I show that to some extent the model matches the level and its cyclical behavior of the US income distribution. The simulated income distribution features large income shares in top income groups and small income shares in bottom income groups as shown in the Current Population Survey data. As for the cyclical property of income distribution, the simulated result qualitatively matches most of the correlations of income shares with output and 95/50 and 50/20 ratios.

I apply the model to carry out a policy experiment in which the government obeys specific policy rules and chooses the parameter values indicating responses of capital tax rate and debt to output. I first calibrate these parameters to the US fiscal policy data as the baseline. Then I evaluate different combinations of counterfactual fiscal policy instruments. I find that in my heterogeneous-agent model the choice of capital tax policy or debt policy creates a welfare conflict between entrepreneurs and workers. A policy which specifies a high capital tax rate and a low debt level in the recession benefits entrepreneurs while harms workers' welfare. The mean level of bonds rises while the average capital tax rate declines compared to the baseline. More government bonds give entrepreneurs insurance and crowd out capital, yet less tax burden encourages entrepreneurs to invest more in capital. The policy slightly enhances the average capital stock and the average wage rate. The mean consumption of entrepreneurs increases due to less tax burden and higher asset income. The government budget balance requires a higher average labor tax rate, which causes a decrease in the mean consumption of workers. Meanwhile, the volatility of consumption of entrepreneurs greatly increases under the policy combination. Yet for entrepreneurs, the increase in the mean consumption outweighs the welfare loss from a higher volatility. Workers see less mean consumption and a higher volatility. Thus, entrepreneurs are better off and workers are worse off.

I ask when the government should tax capital more and issue more debt, and in particular, whether to do it during a boom or a bust. In the classical business cycle framework, the seminal paper by Chari, Christiano and Kehoe (1994) shows that the correlation of an optimal capital income tax rate with technology shock is negative with uncontingent debt in their baseline model. In addition, when there is a negative innovation to the technology shock or a positive innovation to government consumption, there is a positive innovation in the tax on private assets. My result on fiscal policy seems to confirm their finding: The policy that maximizes the utilitarian social utility features an increase by 0.45 percent in the capital tax rate and a decrease by 0.59 percent in the debt level as the output drops by one percent. Nonetheless, the welfare conflict indicates that workers prefer a capital tax rate which increases less in the recession. The socially preferred policy may reduce the capital tax rate in the bust if the government, instead of adopting the utilitarian welfare criterion, puts more weight on workers. Therefore, it is possible to overturn the result of Chari, Christiano and Kehoe (1994) such that the government should cut the capital income tax rate in the recession.

I study the impulse responses of aggregate variables to a standard deviation of negative aggregate productivity shock under the baseline policy combination targeted to the data and the other three policy combinations optimizing entrepreneurs, workers and all agents, respectively. These policy combinations affect the behaviors of aggregate variables after the shock. In general, the higher capital tax rate and the lower debt when the adverse shock hits, the more capital and output in the early stage after the shock. However, the recovery of capital slows down and falls behind the counterparts under other policy combinations because the average return to capital is lower. This policy also reduces the consumption of entrepreneurs most in the beginning and cause the slowest recovery. Under the high capital tax and the low debt level, the labor tax rate rises, which lowers the consumption of workers more than other policies. Yet since the output and the wage recover faster, so does the consumption of workers.

To see if my results are robust to the choice of the policy instrument that balances the government budget, I keep the labor tax rate constant and study the cyclical property of debt on welfare with the capital tax rate to balance the government budget. I find that the welfare conflict still occurs in this experiment. Entrepreneurs prefer a low debt level in the recession while workers prefer a high level. Compared to the case of the time-varying labor tax rate, the policy that maximizes workers' welfare demands a lower volatility of debt. The utilitarian social utility requires a debt policy close to the one maximizing workers' welfare.

I examine whether the result of the welfare conflict is robust to constant volatility of idiosyncratic investment risks and no idiosyncratic investment risks. I find that the welfare conflict holds with constant volatility of idiosyncratic investment risks, but fails to exist under reasonable capital tax policies without these risks.

My paper contributes to the literature of uninsured idiosyncratic investment risks, which has demonstrated theoretically that idiosyncratic investment risks influence the steady state of the aggregate economy or on the stationary distribution of income, earnings or wealth. These studies find that market incompleteness may lead to underaccumulation or overaccumulation of capital, depending on the parameterization (Angeletos (2007); Angeletos and Calvet (2006); Covas (2006); Meh and Quadrini (2006)). Other scholars apply the framework with idiosyncratic investment risks to explain questions about fiscal policies (Angeletos and Panousi (2009), Panousi (2010)), international difference of growth (Angeletos and Panousi (2011) and the like. Since aggregate shocks are missing, their studies are silent about the cyclical behavior or the welfare change over the business cycle. My paper, on the contrary, adds aggregate shocks to the framework of Angeletos (2007). The classic Krusell and Smith (1998) framework requires the entire wealth distribution as a state variable. Since the wealth distribution is an infinite-dimensional object, the computation of such a model with aggregate risks is difficult. My model provides a tractable tool to analyze the fluctuations of aggregate economy and income distribution and assess policies over the business cycle under the setting of heterogeneous agents because it allows for exact aggregation, or equivalent to say, I can find a representative agent to study the behaviors of all agents.

This paper is closer to the recent studies embedding aggregate shocks into the framework with idiosyncratic investment risks. Goldberg (2014) sets up a framework with idiosyncratic investment risks in the business cycle. My paper differs from his in a few ways. First, I model incomplete markets with a unique non-state-contingent bond being the only security in the market; Goldberg provides state-contingent promises but with moral hazard. Second, he aims to build on a theoretical framework which incorporates uninsured idiosyncratic investment risks, aggregate shocks and borrowing constraint while I build up this model to help answer questions about policy analysis. Another example is the work of Nezafat and Slavík (2015), who construct a production-based asset pricing model with aggregate risks, idiosyncratic investment risks and financial frictions to explain the high volatility of asset prices. I, instead, talk about welfare and the assessment of fiscal policies.

The paper also belongs to the literature on fiscal policies. These studies usually either assume a representative agent and explore the effect of policies on the macroeconomy, neglecting the distributive issues; or they do consider heterogeneity but exclude aggregate risks, which makes them unable to discuss how effectively a government modifies fiscal policies to improve welfare when witnessing aggregate economic fluctuations. The exceptions are Werning (2007) and Bassetto (2014). Both employ complete markets to simplify the solution to the optimal policy problem. In contrast, I build up the problem in incomplete markets, in line with empirical evidence. In addition, Werning (2007) focuses on whether to smooth the tax rate while Bassetto (2014) builds up a model without capital so that the model is unable to assess the capital income tax. I focus on when to tax capital and issue debt based on the cyclicality of fiscal policies.

The rest of the paper is organized as follows. I extend Angeletos' framework to model business cycles in Section 2 and characterize equilibrium in Section 3. In Section 4, I quantitatively study the income shares of different income groups and their cyclical properties generated by my model. I also make a comparison to the data. Section 5 carries out experiments on fiscal policies. Section 6 examines the robustness of policy evaluation with constant idiosyncratic investment risks and without idiosyncratic investment risks. Section 7 concludes.

2 Angeletos' Framework over Business Cycles

2.1 Economy

The economy is populated with two types of agents: entrepreneurs and workers. I normalize the measure of entrepreneurs to 1 and the measure of workers by λ. I interpret λ as the ratio of numbers of workers over entrepreneurs or the worker-entrepreneur ratio. There is a continuum of individuals for each type, indexed by i and j, respectively. Each worker is endowed with $\frac{1}{\lambda}$ unit of time so that the aggregate labor endowment is 1. I denote u_t as instantaneous utility; each agent maximizes her expected lifetime utility subject to her own budget constraint and borrowing constraints. The aggregate productivity z_t affects the production of each firm in the economy and follows an AR(1) process

$$\log z_{t+1} = \rho_z \log z_t + \epsilon^z_{t+1}, \tag{1}$$

where ϵ^z_{t+1} is normally distributed, $\epsilon^z_{t+1} \sim \mathcal{N}(0, \sigma^2_z)$, and the autoregressive parameter $\rho_z \in [0, 1)$.

2.2 Entrepreneurs

I assume that entrepreneurs only care about consumption. I specify a CRRA utility function $u(c^i_t) = \frac{(c^i_t)^{1-\gamma}}{1-\gamma}$, where γ denotes the degree of risk aversion. Each entrepreneur owns a private firm. At t, an entrepreneur can invest capital k^i_{t+1} in the firm owned by herself, but not in other private firms; she can purchase riskfree government bonds b^i_{t+1} as an alternative financial asset. Entrepreneurs do not work and receive asset gains as the unique source of income. r^i_t and R_t denote the capital rental rate to her firm, and the interest rate of bonds, respectively, while τ^a_t, the capital tax rate. The effective wealth consists of gross income from capital and bond holdings net of taxation. The budget constraint and nonnegativity constrains are

$$c_t^i + k_{t+1}^i + b_{t+1}^i = [(1-\tau_t^a)(r_t^i - \delta) + 1]k_t^i + [(1-\tau_t^a)(R_t - 1) + 1]b_t^i, \quad (2)$$

$$c_t^i \geq 0 \text{ and } k_{t+1}^i \geq 0.$$

Entrepreneurs are allowed to borrow in the bond market but the borrowing amount has to fulfill the No-Ponzi condition

$$\lim_{T\to\infty} \mathbb{E}_t \frac{b_{t+T}^i}{\prod_{s=0}^{T-1} [(1-\tau_{t+s}^a)(R_{t+s} - 1) + 1]} = 0. \quad (3)$$

2.3 Firms

Each firm hires labor in a completely competitive labor market, employs its owner's capital and produces consumption goods. I name the firm run by the entrepreneur i also with i. I assume the neoclassical production technology

$$y_t^i = F(k_t^i, n_t^i, A_t^i) = (A_t^i)^\alpha (k_t^i)^\alpha (n_t^i)^{1-\alpha},$$

which exhibits constant returns to scale with respect to k and n. The firm-level productivity specific to firm i, A_t^i affects the final output y_t^i. Particularly, A_t^i consists of two components, an idiosyncratic production risk e_t^i and aggregate productivity z_t: $\log A_t^i = \log z_t - \frac{\sigma_{e,t}^2}{2} + e_t^i$, where e_t^i is independently and identically distributed among firms and across time while z_t captures the business cycle. The idiosyncratic risk e_t^i is modelled as a normal, $e_t^i \sim \mathcal{N}(0, \sigma_{e,t}^2)$, where $\sigma_{e,t}$ represents the standard deviation of idiosyncratic investment risks at t, which may vary across periods following $\sigma_{e,t}^2 = \sigma_e^2 \exp(-\eta \log z_t)$. η denotes the response of volatility to the cycle. Bloom et al (2014) measure the dispersion of TFP shocks for a panel of plants and find that the volatility of firm-specific productivity rises during the recession. The idiosyncratic risk captures the position of a specific firm ranked by the firm-level productivity. The correction term $-\frac{\sigma_{e,t}^2}{2}$ renders the average productivity of private firms equivalent to the aggregate productivity. I define the profit of firm i as the firm revenue net of labor costs

$$\pi_t^i(k_t^i, n_t^i, A_t^i) = y_t^i - w_t n_t^i,$$

where w_t represents the wage rate at t. The competitive labor market ensures a universal wage rate in each period and the wage depends on the aggregate productivity and aggregate allocations.

The assumption of i.i.d idiosyncratic investment risks may seem restrictive. Nevertheless, it is necessary for tractability. Also, it may not be that unrealistic given that DeBacker et al (2013) find that business income is much less persistent than labor income and it is characterized by higher probabilities of extreme upward or downward mobility. For instance, conditional on leaving the starting business income decile, a household faces a 52% probability of moving to either of the two immediately adjacent deciles over a year. More strikingly, households starting at the lowest decile of the business income distribution face a 12% probability of transitioning to decile 8 or higher, whereas the corresponding probability is essentially zero for labor income. Hence, business income may transition from low to high amounts within a relatively short time and i.i.d shock is a reasonable abstraction from the fact.

2.4 Workers

Worker j has preferences on consumption, which is specified as $u(c_t^j) = \frac{(c_t^j)^{1-\gamma}}{1-\gamma}$. I assume that a worker shares the same curvature of consumption as an entrepreneur. Workers supply their labor in the competitive labor market, work in firms owned by entrepreneurs and consume all of their earnings in the manner of hand-to-mouth agents. This is not unrealistic given that quite a number of households hardly own any wealth other than a house. Workers provide identical working hours but differ in their labor efficiency e_t^j, which is independently and identically distributed across workers. A worker's personal labor income depends on idiosyncratic labor efficiency and the wage rate. A worker is taxed by a proportional labor tax rate of τ_t^n. The assumption of hand-to-mouth implies that the aggregate consumption and income for workers do not depend on the distribution of workers. I assume that idiosyncratic labor efficiency follows a persistent stochastic process,

$$\log e_{t+1}^j = \rho_w \log e_t^j + \epsilon_{t+1}^j, \tag{4}$$

where $\rho_w \in [0, 1)$ denotes the autocorrelation of labor efficiency and ϵ_{t+1}^j is normally distributed, $\epsilon_{t+1}^j \sim \mathcal{N}(0, \sigma_w^2)$.

The budget constraint for worker j is

$$c_t^j = (1 - \tau_t^n) w_t \frac{1}{\lambda} e_t^j. \tag{5}$$

2.5 Government

Government spending, g_t, is assumed to be exogenous, following an AR(1) process.

$$\log g_{t+1} = (1 - \rho_g) \log \bar{g} + \rho_g \log g_t + \epsilon_{t+1}^g, \tag{6}$$

where the steady state level of government consumption is $\bar{g}$, the autoregressive parameter $\rho_g \in [0, 1)$ and ϵ_{t+1}^g is normally distributed, $\epsilon_{t+1}^g \sim \mathcal{N}(0, \sigma_g^2)$.

Each period the government finances government spending by levying the proportional taxes $\{\tau_t^n, \tau_t^a\}$ and issuing bonds $\{B_{t+1}\}$.

$$\int_i \tau_t^a [(r_t^i - \delta) k_t^i + (R_t - 1) b_t^i] + \int_j \tau_t^n w_t \frac{1}{\lambda} e_t^j + B_{t+1} = g_t + R_t B_t, \tag{7}$$

where B_{t+1} denotes the total amount of bonds issued at t and paid off at $t + 1$. I assume that the government does not distinguish the gains from risky assets and from riskfree assets so that it taxes the asset income at the same rate. In this paper, I intend to analyze the optimal policies of capital taxation and debt conditional on the long-run levels, or the effect on the social welfare of cyclical properties of taxation and debt. Thus I assume that fiscal policies τ_t^a and B_{t+1} are functions of log-deviation of output, indicating that the government adjusts its policies in the cycle.

$$\log\left(\frac{\tau_t^a}{\bar{\tau}^a}\right) = m_{Y\tau}\log\left(\frac{Y_t}{\bar{Y}}\right); \tag{8}$$

$$\log\left(\frac{B_{t+1}}{\bar{B}}\right) = \rho_B \log\left(\frac{B_t}{\bar{B}}\right) + m_{YB}\log\left(\frac{Y_t}{\bar{Y}}\right) \tag{9}$$

$\bar{\tau}^a$ represents the steady state level of tax rate, $\bar{Y}$ denotes the steady state value of output and $\bar{B}$ shows the steady state level of debt. $m_{Y\tau}$ and m_{YB} are two coefficients governing the responses of current tax rates and debt to output fluctuations, respectively. ρ_B denotes the autoregressive property of government debt[1]. For instance, the government taxes more capital income and issues more government bonds during recessions if $m_{Y\tau} < 0$ and $m_{YB} < 0$. In a word, the government sets the rules of debt and capital tax, and applies labor tax to balance the budget.

2.6 Timing

In every period, aggregate shocks and idiosyncratic risks first hit the economy. The government then announces fiscal policies based on the current state. After observing the shocks and policies, workers supply labor while the firm optimally chooses the demand of labor. The firm takes labor and predetermined capital as inputs to produce. The entrepreneur consumes, accumulates capital and purchases bonds after she receives income from interest and bond payment, and pays taxes. The worker consumes all the after-tax labor income.

2.7 Stationarity

The current model would imply a nonstationary distribution of wealth and income. I add an exogenous death probability Pr_d equal to all entrepreneurs to guarantee limited income by entrepreneurs. Specifically the death shock is assumed to happen after every entrepreneur makes her own decision each time. Some entrepreneurs die while the same number of newborns enter into the economy. Then every newborn inherits the average amount of risky and riskfree assets previously owned by the dead. I will show in the later section that entrepreneurs make saving and portfolio choices independent of the income distribution, only as functions of the aggregate state. Hence this assumption does not change the original choices before the death and facilitates the computation.

3 Equilibrium

3.1 Equilibrium Definition

Denoting K_t, B_t, z_t, g_t as the aggregate state s_t. I define an equilibrium as a stochastic sequence of prices $\{w_t(s_t), R_{t+1}(s_t)\}_{t=0}^{\infty}$, a stochastic sequence of individual allocations $\{c_t^i(k_t^i, b_t^i, e_t^i; s_t), k_{t+1}^i(k_t^i, b_t^i, e_t^i; s_t), b_{t+1}^i(k_t^i, b_t^i, e_t^i; s_t)\}_{t=}^{\infty}$

[1] I set the autoregressive specification of debt because the government has to stabilize the debt holding. As for tax policy, I choose a simple version of Leeper's specification to facilitate my study.

$i \in [0,1]$, for entrepreneurs and $\{c_t^j(e_t^j;s_t), n_t^j(e_t^j;s_t)\}_{t=0}^{\infty}$, $j \in (1, \lambda{+}1]$, for workers, $\{y_t^i(k_t^i, e_t^i; s_t), n_t^i(k_t^i, e_t^i; s_t)\}_{t=0}^{\infty}$ for firms, and aggregate allocations $\{C_t^E(s_t), C_t^W(s_t), K_{t+1}(s_t), Y_t(s_t)\}_{t=0}^{\infty}$, such that

(1) Given prices $\{w_t(s_t), R_{t+1}(s_t)\}_{t=0}^{\infty}$, the fiscal policy $\{\tau_t^n(s_t), \tau_t^a(s_t), B_{t+1}(s_t)\}$ and the distribution of initial assets k_0^i and b_0^i, every entrepreneur i and every worker j maximize their respective lifetime utility by choosing $\{c_t^i(k_t^i, b_t^i, e_t^i; s_t), k_{t+1}^i(k_t^i, b_t^i, e_t^i; s_t), b_{t+1}^i(k_t^i, b_t^i, e_t^i; s_t)\}_{t=0}^{\infty}$ and $\{c_t^j(e_t^j;s_t), n_t^j(e_t^j;s_t)\}_{t=0}^{\infty}$, and every firm i maximizes its profit by choosing $\{n_t^i(k_t^i, e_t^i; s_t), y_t^i(k_t^i, e_t^i; s_t)\}$.

(2) Aggregation: $C_t^E(s_t) = \int_i c_t^i(k_t^i, b_t^i, e_t^i; s_t)$, $Y_t(s_t) = \int_i y_t^i(k_t^i, e_t^i; s_t)$, $K_{t+1}(s_t) = \int_i k_{t+1}^i(k_t^i, b_t^i, e_t^i; s_t)$ and $C_t^W(s_t) = \int_j c_t^j(e_t^j; s_t)$ for all t.

(3) Labor market clearing: $\int_i n_t^i(k_t^i, e_t^i; s_t) = \int_j n_t^j(e_t^j; s_t) e_t^j$ for all t.

(4) Bond market clearing: $\int_i b_{t+1}^i(k_t^i, b_t^i, e_t^i; s_t) = B_{t+1}(s_t)$ for all t.

(5) Goods market clearing: $C_t^E(s_t) + C_t^W(s_t) + K_{t+1}(s_t) + g_t = Y_t(s_t) + (1-\delta)K_t(s_{t-1})$.

(6) The government budget constraint holds given capital tax and bond specifications for all t, i.e. (7), (8) and (9) hold.

For the expository purpose, I will mostly drop the state in each variable for the rest of my paper and only use the expression with the state when it facilitates the analysis.

3.2 Individual Behavior

Because the firm chooses employment n_t^i after observing the shock and after determining the capital stock, n_t^i is the only control variable to maximize the profit. By constant returns to scale, optimal firm employment and capital income are linear in capital, as in Angeletos (2007):

$$n_t^i = \left(\frac{1-\alpha}{w_t}\right)^{\frac{1}{\alpha}} A_t^i k_t^i = n(A_t^i, w_t) k_t^i, \tag{10}$$

$$\pi_t^i = \left[\alpha \left(\frac{1-\alpha}{w_t}\right)^{\frac{1}{\alpha}-1} A_t^i\right] k_t^i = r(A_t^i, w_t) k_t^i. \tag{11}$$

It indicates that the firm experiences linear returns to investment by adjusting its employment linearly to the capital stock. Notice that the return to a private firm i, $r_t^i = r(A_t^i, w_t)$, is expressed by the firm specific productivity A_t^i and the economy-wide wage which can be further expressed by the aggregate productivity and total capital stock.

Denote the effective wealth of entrepreneur i in period t by

$$x_t^i \equiv \left[(1-\tau_t^a)(r(A_t^i, w_t) - \delta) + 1\right] k_t^i + [(1-\tau_t^a)(R_t - 1) + 1] b_t^i.$$

I rewrite the budget constraint as $c_t^i + k_{t+1}^i + b_{t+1}^i = x_t^i$.

I characterize the entrepreneur's behavior following Angeletos (2007) in Lemma 1, whose proof is provided in Appendix A.

Lemma 1 *Given prices and fiscal policies, optimal consumption, capital stock in the private firm and in* *the corporate firm, and bond holdings are linear in effective wealth as shown in (13), (14) and (15);*

$$c_t^i = \nu_t x_t^i, \tag{12}$$

$$k_{t+1}^i = (1-\nu_t)\phi_t x_t^i, \tag{13}$$

$$b_{t+1}^i = (1-\nu_t)(1-\phi_t)x_t^i, \tag{14}$$

where the marginal propensity to consume out of effective wealth ν_t, and the share of private equity in the portfolio ϕ_t are two stochastic coefficients, depending solely on the current aggregate states, s_t, and satisfying

$$\phi_t = \arg\max_{\phi\in[0,1]} \mathbb{CE}_t \left\{\phi(r(A_{t+1}^i, w_{t+1}) - \delta) + (1-\phi)(R_{t+1}-1)\right\}, \tag{15}$$

$$\begin{aligned}\nu_t^{-\gamma} = \beta_s(1-\nu_t)^{-\gamma}\mathbb{E}_t \big\{\nu_{t+1}^{-\gamma}\{\phi_t[(1-\tau_{t+1}^a)(r(A_{t+1}^i, w_{t+1}) - \delta) + 1] + \\ +(1-\phi_t)[(1-\tau_{t+1}^a)(R_{t+1}-1)+1]\}^{1-\gamma}\big\},\end{aligned} \tag{16}$$

where $\mathbb{CE}$ represents the certainty equivalent of an entrepreneur.[2]

Define the value function for entrepreneurs as $V(x_t^i)$ which is given by

$$V(x_t^i) = \frac{\nu_t^{-\gamma}(x_t^i)^{1-\gamma}}{1-\gamma}. \tag{17}$$

The entrepreneur makes the optimal portfolio choice ϕ_t by maximizing risk-adjusted portfolio returns, expressed by the certainty equivalent $\mathbb{CE}$ of the portfolio return given the saving choice $(1-\nu_t)$. If the return of capital is surely greater than the return of bonds, she will invest all her savings on capital. The uncertainty of capital return, though, induces her to divide her investment on both assets, which ensures an interior point of portfolio decisions. The current specification of tax policies implies that a change in the capital tax rate fails to directly influence the portfolio choice since both types of assets are taxed at the same rate. Nonetheless, the capital tax rate distorts the saving choice, indirectly affecting asset returns and the composition of portfolio. Given the portfolio choice, the entrepreneur chooses the marginal propensity to consume according to the intertemporal condition to maximize the life utility. Because of i.i.d idiosyncratic investment risks and homogeneity of the production and utility functions, entrepreneurs make the saving and portfolio decisions independent of one period's income distribution. Hence the marginal propensity to consume and the risky-asset-over-wealth ratio only depend on the aggregate state.

The hand-to-mouth worker consumes all her after-tax labor income every period. So the consumption path is completely characterized by the budget constraint of workers.

3.3 General Equilibrium

Recall that aggregate productivity z_t and idiosyncratic risks e_t^i are orthogonal. Aggregate labor demand and firm profit are given by $N_t^D = \tilde{n}(w_t, z_t)K_t$ and $\Pi_t = \tilde{r}(w_t, z_t)K_t$, where $\tilde{n}(w_t, z_t) \equiv \int_{-\infty}^{\infty} n(A_t^i, w_t)\, \mathrm{d}F(e_t^i) =$

[2] I denote $\beta_s = \beta(1-Pr_d)$

$\left(\frac{1-\alpha}{w_t}\right)^{\frac{1}{\alpha}} z_t$ and $\tilde{r}(w_t, z_t) \equiv \int_{-\infty}^{\infty} r(A_t^i, w_t)\ \mathrm{d}F(e_t^i) = \alpha \left(\frac{1-\alpha}{w_t}\right)^{\frac{1}{\alpha}-1} z_t$. $F(\cdot)$ represents the distribution function of i.i.d idiosyncratic investment risks. Thus $\tilde{n}(w_t, z_t)$ denotes the average labor employed by a firm and $\tilde{r}(w_t, z_t)$, the average capital return to a private firm. Let $N_t^S = \int_j \frac{1}{\lambda} e_t^j = 1$ represent aggregate labor supply; labor-market clearing requires that $N_t^D = \tilde{n}(w_t, z_t) K_t = 1$. The wage is expressed as $w_t = w(K_t, z_t) = (1-\alpha)\,(z_t K_t)^{\alpha}$. Further algebra shows that $r_t = \tilde{r}(w_t, z_t) = \alpha z_t^{\alpha} K_t^{\alpha-1}$. I write aggregate output as $Y_t = \Pi_t + w_t N_t^S = r_t K_t + w_t = z_t^{\alpha} K_t^{\alpha}$. Aggregate allocations are also independent of the wealth distribution because consumption, bond holdings, and private investment are linear in individual wealth and idiosyncratic risks at one period are i.i.d across firms and periods. The general equilibrium is determined by the following equations, where I drop the determination of z_t and g_t as they are assumed to be exogenous. I denote C_t^E as the aggregate consumption assigned to the group of entrepreneurs and C_t^W, to the group of workers.

$$C_t^E + C_t^W + K_{t+1} + g_t = z_t^{\alpha} K_t^{\alpha} + (1-\delta) K_t, \tag{18}$$

$$C_t^W = (1-\tau_t^n) w_t, \tag{19}$$

$$C_t^E = \nu_t \{[(1-\tau_t^a)(r_t - \delta) + 1] K_t + [(1-\tau_t^a)(R_t - 1) + 1] B_t\}, \tag{20}$$

$$K_{t+1} = (1-\nu_t)\phi_t \{[(1-\tau_t^a)(r_t - \delta) + 1] K_t + [(1-\tau_t^a)(R_t - 1) + 1] B_t\}, \tag{21}$$

$$B_{t+1} = (1-\nu_t)(1-\phi_t) \{[(1-\tau_t^a)(r_t - \delta) + 1] K_t + [(1-\tau_t^a)(R_t - 1) + 1] B_t\}, \tag{22}$$

These five equations, together with (16) and (17) solve for seven variables C_t^E, C_t^W, K_{t+1}, B_{t+1}, R_{t+1}, ν_t and ϕ_t, given the current aggregate state and the fiscal policy.

3.4 Computation Process

The framework allows me to separate the computation of aggregate variables and income distribution. First, I apply the 5th-order approximation conducted by Dynare to simulate a time series of equilibrium at the aggregate level in the business cycle starting from the steady state. I am able to accomplish it because, economically speaking, every entrepreneur makes exactly identical saving and portfolio decisions regardless of their individual wealth. In another word, I can always find a representative entrepreneur to study the behaviors of all entrepreneurs. Then I divide aggregate allocations into the individual level. The next period's capital and bonds of entrepreneur i can be calculated as a product of saving choice, portfolio choice and effective wealth as Equation (14) and (15). The first two components are obtained from the time series of general equilibrium. I separate the last part into idiosyncratic capital returns, interest rate of bonds and capital and bonds stock held from last period. The time series of equilibrium contains the values of interest rate and the average capital returns, then from (35) we get idiosyncratic capital returns by inputting a stochastic process of idiosyncratic risks. Thus the amount of assets forms a recursion given asset prices in the aggregate level and idiosyncratic risks. Workers share the total labor income determined by the general equilibrium with different labor efficiency.

I simulate 2000 entrepreneurs and as a consequence 10000 is the number of workers. I allocate equally the steady state level of capital and bonds to 2000 entrepreneurs as the initial point. Since individual capital returns are idiosyncratic to entrepreneurs, lucky entrepreneurs accumulate more and more assets so that the gap of capital income rises over time. Yet the assumption of random death limits infinite inequality. I record the income of every agent, including entrepreneurs and workers, order all values and then compute the quantile statistics and the like. I run a simulation with 2500 periods and drop the first 1500 periods to guarantee stability.

4 Business Cycle Statistics of the Income Distribution

This section shows that the model, with aggregate shocks, can mimic the income distribution and its cyclicality at a small computational cost. The model matches the dynamics of the income distribution shown in the data qualitatively for most the quantiles.

4.1 Calibration

Numerical analysis is needed to evaluate the effect of idiosyncratic investment risks on the steady state. I first describe the details of calibrating the model in this subsection.

I assume that a period in my model corresponds to a year. I use some parameter values which are common in the literature of business cycle: The capital share of output, α, is assumed to be 0.36; the depreciation rate, δ, 0.08.

The probability of death, Pr_d, for entrepreneurs is calibrated to the working lifetime of an entrepreneur or a private firm which is unclear and hard to measure. I assume the probability of death to be 0.025 for an expected working lifetime of 40 years. I then calibrate the discount rate so that the subjective discount rate of entrepreneurs after considering death produces a realistic capital-output ratio, 2.7 as in the US. I assume the worker-entrepreneur ratio as 5 to match the estimation of business owners and self-employed in the US reported by Cagetti and De Nardi (2006).[3]

The choice of risk aversion degree matters for my study, especially when I evaluate fiscal policies. The baseline calibration sets the risk aversion degree equal to 2. The reason is that to assess fiscal policies requires the welfare comparison and the value is commonly applied in the literature on welfare.

I apply the effective marginal tax rate of the US computed by Devereux, Lockwood and Redoano (2008) as the capital tax rate in my model. I take the mean of these rates from 1979 to 2005 and the

[3]The value seems much higher than the estimated proportion of hand-to-mouth workers. For instance, Kaplan, Violante and Weidner (2014) define a household to be hand-to-mouth in a period if it consumes all its cash-on-hand for the period and carries zero liquid wealth between the current and next period. They conclude that on average, 31 percent of US households are hand-to-mouth from 1989 to 2010. The assumption of hand-to-mouth workers in this paper aims to shut down saving from workers who face labor income risks to simplify the analysis and the computation. Since the model does not involve housing, I focus on the individuals who have no non-housing wealth defined by net worth. About 60% of households in the US own no wealth other than their home according to the 2007 Survey of Consumer Finances. Although my choice of the worker/entrepreneur ratio still overstates the composition of hand-to-mouth workers in this sense, I have a reason as follows. The 2016 poll by The Associated Press-NORC Center for Public Affairs Research uncovers that two thirds of Americans would have difficulty coming up with the money to cover a 1000-dollar emergency. In a 2015 study by the Federal Reserve, 47 percent of respondents said they either could not cover even a 400-dollar emergency expense or would have to sell something or borrow money. This paper mainly investigates the fluctuations of income distribution in the business cycle. When a sufficiently negative shock happens, individuals probably face a much larger loss than 1000 dollars so that the assumption of more than 80 percent of hand-to-mouth workers in my model does not exaggerate much the situation of US citizens in the business cycle.

value is 22.19%, which is close to the capital tax rate used by Chari, Christiano and Kehoe (1994). I pick the labor income tax rate to balance the government budget. In the benchmark calibration, I set the steady state levels of debt as 60 percent of output and government consumption as 18 percent of output, both of which depict the U.S. situation before the recent crisis.

It is hard to find an exact measurement of idiosyncratic investment risks in the empirical studies. DeBacker et al (2012) find that the standard deviation of uninsurable idiosyncratic income risk from privately held businesses accounts for 45 percent of the average business income.[4] Therefore I calibrate my model to match their finding; specifically the cross sectional standard deviation of individual firm's return is 45% of the average return. Notice that a number of macroeconomic studies, such as Sandri (2014), apply the volatility of firms' returns as 50% of the mean return, thus I assure my calibration close to their choice and my results comparable.

I set the autocorrelation of idiosyncratic labor income risks equal to 0.9989 and the standard deviation, 0.0166 using the estimation of Storesletten, Telmer and Yaron (2004).

The calibration for the exogenous technology process and government spending follows Chari, Christiano and Kehoe (1994). I use the FRED dataset and the OECD tax dataset to pin down the responses of capital tax and debt to output in the above specified fiscal policies. I regress debt on the HP-filtered log of output and lagged debt following the fiscal policy specification. I estimate the response of capital tax rate to output deviation with a similar method except that the regressor only contains HP-filtered log of output. Worth to mention, the tax policy reduces (amplifies) the business cycle effect by taxing less (more) in the recession (expansion). Meanwhile, the government reduces debt in the expansion and vice versa.

Recall that I correlate the volatility of idiosyncratic investment risks with aggregate productivity, $e_t^i \sim \mathcal{N}(0, \sigma_e^2 \exp(-\eta \log z_t)$. I calibrate the sensitivity of the variance of idiosyncratic investment risks to aggregate productivity shocks, η, to match the empirical finding of Bloom et al (2014) that plant-level TFP shocks increased in variance by 76% during the recent recession (2008 to 2009) compared to the years before the recession (2005 to 2006). I plug the deviation of the logarithm of TFP from the trend in 2006 and in 2009 into the expression of the variance of idiosyncratic investment risks, $\sigma_e^2 \exp(-\eta \log z_t)$ and back up the value of η. All above parameter values for the benchmark calibration are included in Table 1.

4.2 Cyclical Behavior of Aggregate Variables in General Equilibrium

Before presenting the results on the income distribution, I briefly show the correlations of different equilibrium variables to output in my baseline model in the following table. Particularly, I emphasize the marginal propensity to consume, ν, and the portfolio choice, ϕ, since these two variables summarize the key property of the entire equilibrium allocations and prices.

The marginal propensity to consume, ν, is positively correlated with output. Investors tend to save more assets in the recession because highly countercyclical idiosyncratic investment risks amplify the

[4] They employ individual income tax returns data from the Internal Revenue Service over 23 years, compute for each household the time series standard deviation of its business income normalized by the household's average total income over time, and then combine those business income "coefficients of variation" into one cross sectional average.

Table 1: Calibration for the benchmark case

Parameters	**Values**	**Descriptions**
α	0.36	capital share of output
β	0.98	subjective discount rate
Pr_d	0.025	probability of death for entrepreneurs
δ	0.08	depreciation rate
γ	2	risk aversion degree
$\bar{b}$	60% output	steady state level of bonds
$\bar{g}$	18% output	steady state level of government consumption
$\bar{\tau}^a$	22.19%	steady state asset income tax rate
σ_e	45% average return	standard deviation of idiosyncratic risks
λ	5	worker/entrepreneur ratio
ρ_w	0.9989	autocorrelation of labor efficiency
σ_w	0.0166	cross-sectional standard deviation of labor income
η	11.10	sensitivity of the variance of idiosyncratic production risks to productivity shocks
ρ_z	0.81	autocorrelation of technology process
σ_z	0.05	standard deviation of technology shock
ρ_g	0.89	autocorrelation of government spending
σ_g	0.07	standard deviation of government spending shock
ρ_B	0.84	autocorrelation of government debt
m_{YB}	-0.62	response of debt to output gap
$m_{Y\tau}$	0.91	response of tax to output gap

Table 2: Cyclical Behavior of Aggregate Variables

Variables	**Correlation with Output**
ν	0.61
ϕ	0.84
Capital	0.85
Output	1
Average Rental Rate of Capital	0.14
Interest Rate	0.86
Risk Premium	-0.74
Wage	1
Ent. Con.	0.99
Wor. Con.	0.59

precautionary saving motive. In addition, they invest more in safe assets so that the capital share in the portfolio, ϕ, shows procyclicality, which causes procyclical capital stock.

The cyclical behavior of interest rates is related to the saving and portfolio choices. Agents demand more riskfree assets in the bust; meanwhile, the government supplies more bonds according to the policy specification. A smaller increase in supply relative to the change demand results in a higher equilibrium price of government bonds, or equivalently, a lower interest rate. Average rental rate of capital and other allocations follow the change of productivity. The risk premium rises, although the average rental rate of capital and interest rate both decline in the recession.

4.3 Income Inequality: Level

The income of an entrepreneur i is defined as the asset gains, $I_t^i = (r_t^i - \delta)k_{e,t}^i + (R_t - 1)b_t^i$ as Castañeda et al (1998) do, while the income of a worker j is the labor income, $\frac{1}{\lambda} w_t e_t^j$. My model does a decent job in generating income equality as the data show. For comparison, I take the statistics of the US income distribution from Current Population Survey and compute the means of income shares over time. Next column reports the means of income shares of different income groups obtained from the simulation. My model fits the data in the sense that it generates the thick tails in both ends of the income distribution although the rich own less income and the poor share more compared with the data.

Table 3: Income Inequality: Level

Quantiles	**Share of income (Data)**	**Share of income Model**
1st (bottom 20%)	4.1%	6.62%
2nd (20-40%)	10.3%	14.80%
3rd (40-60%)	16.4%	18.35%
4th (60-80%)	23.9%	22.65%
5th (80-95%)	27.0%	22.47%
Top 5%	18.4%	15.10%
Ratios	**Value (Data)**	**Value (Model)**
95/50 ratio	3.29	2.06
50/20 ratio	2.40	1.42

All the values of income shares are means computed using corresponding data from CPS from 1947 to 2013. CPS provides the statistics of households only since 1967 so I pick up the income share of families from 1947 to 1966.

Entrepreneurs account for the majority in the top income group and for over one third of agents in the bottom as shown in Table 4. It follows the dispersed distribution of idiosyncratic investment risks. Most entrepreneurs are rich in the model, yet some unlucky ones find themselves in the low income groups. Since entrepreneurs in the benchmark calibration are in line with the definition of business owners and self-employed, the simulation generates poor entrepreneurs as in reality since part of self-employed have low income. It indicates that the thick tails in both ends are obtained by modelling entrepreneurs with relatively high idiosyncratic investment risks, which is confirmed by other research, such as Benhabib, Bisin and Zhu (2014) and Nirei and Aoki (2014). In addition, I assume hand-to-mouth workers, which creates heterogeneity in the behaviors of agents and leads to inequality.

Table 4: Proportion of Entrepreneurs

Quantiles	**Model**
1st (bottom 20%)	35.77%
2nd (20-40%)	6.07%
3rd (40-60%)	5.89%
4th (60-80%)	7.63%
5th (80-95%)	15.69%
Top 5%	64.79%

4.4 Income Inequality: Cyclical Behavior

Table 5 reports the correlations with output of the income shares owned by income groups in the data and in my model. I choose the same data from CPS as for the level of income inequity and choose seasonally adjusted real GDP measured by 2009 billion dollars during 1948-2013. I detrend the shares, ratios and the log of real GDP by HP-filter and calculate the correlations. For the model, I pick the time series of income shares of the corresponding income groups and obtain the correlations with the output over the business cycle.

Table 5: Correlation of Income Shares with Output

Income groups	**Data**	**Model**
bottom 20%	0.29	0.86
20-40%	0.30	0.96
40-60%	0.15	0.81
60-80%	-0.19	0.40
80-95%	-0.44	-0.60
Top 5%	0.07	-0.88
Ratios	**Data**	**Model**
95/50 ratio	-0.24	-0.88
50/20 ratio	-0.06	-0.88

All the values of income shares use corresponding data from CPS from 1947 to 2013. CPS provides the statistics of households only since 1967 so I pick up the income share of families from 1947 to 1966.

My model with the baseline calibration qualitatively matches correlation with output of income shares qualitatively except the top 5% group and the 60-80% group although the model still differs from the data quantitatively. My model overstates the procyclicality for groups from the bottom to 60% and the countercyclicality for 80-95%.

I wonder why my model presents such quantitative behaviors of the income distribution over the cycle. To uncover the mechanism, I build up two more statistics to measure the qualitative impact of business cycle on individual income: coefficients of variation of $t+1$'s income for entrepreneurs and workers given t's information, $\text{Coef.Var.}_t(I^i_{t+1})$ and $\text{Coef.Var.}_t(I^j_{t+1})$. These two statistics aim to show how an aggregate shock at $t+1$ affects $t+1$'s income for agents who expect it at t. Notice that for simplicity I do not consider the death risk in the subsequent discussion of mechanism since to add death does not affect any qualitative result. I express the two statistics as follows, whose derivation lies in Appendix C.

$$\text{Coef.Var.}_t(I^i_{t+1}) = \frac{\phi_t \alpha z_t^{\alpha\rho_z} K_{t+1}^{\alpha-1}\sqrt{\exp\left[\sigma_e^2 \exp(-\eta\rho_z \log z_t) + \frac{1}{2}(2\alpha - \eta\sigma_e^2)^2\sigma_z^2\right] - \exp\left(\alpha^2\sigma_z^2\right)}}{\phi_t \alpha z_t^{\alpha\rho_z} \exp\left(\frac{1}{2}\alpha^2\sigma_z^2\right) K_{t+1}^{\alpha-1} + (1-\phi_t)(R_{t+1}-1)}. \quad (23)$$

$$\text{Coef.Var.}_t(I^j_{t+1}) = \sqrt{\exp\left(\alpha^2\sigma_z^2 + \sigma_w^2\right) - 1}. \quad (24)$$

The conditional coefficient of variation of workers' next period income is constant. It results from the assumption of hand-to-mouth workers. The wage inherits the cyclical behavior from the output; in particular, its coefficient of variation equates the one of output. Idiosyncratic labor income risks with constant variance enlarge the total variance of workers' income so that eventually the summed volatility appears in the coefficient of variation, indicating the function of both aggregate shocks and idiosyncratic labor income risks on the cross-sectional disparity of workers' income.

When the high productivity is realized, the global wage rises. The mean labor income become higher. The rich workers in the high state experience a larger increase in their income while the poor agents see a smaller increase or even a decrease in the income, depending on the size of idiosyncratic risks. Indeed, almost all workers are highly likely to obtain higher income, though the extent varies, in the high state because idiosyncratic labor income risks have a much smaller dispersion than aggregate shocks in the benchmark simulation. The low productivity produces qualitatively reverse results compared with the preceding ones. Since the coefficient of variation of workers' income remains constant during the cycle, the difference of income in the low state for rich and poor workers stays the same as in the high state. Summarizing both states I conclude that income of all three types of workers shows procyclical behavior.

The conditional coefficient of variation of entrepreneurs' income over the cycle looks more complicated. I start from the discussion on the denominator or roughly speaking, the conditional mean of income. idiosyncratic investment risks, as they are assumed to be i.i.d, exert no effect on next period's expected income. Only aggregate shocks influence the conditional expectation through the channel of returns to assets. When a negative shock happens at t, persistence implies low productivity next period, resulting in smaller share of capital in the portfolio, lower capital stock and lower expected returns to capital. As a consequence, the conditional expected income diminishes during the recession.

When investigating the cyclical property of conditional standard deviation, recall that I assign a positive value to the response of volatility to aggregate productivity η. An adverse aggregate shock causes an increase in the value inside the root operation while the product of other terms outside the root operation goes down. Therefore, The expression cannot easily judge the moving direction of the whole nominator and the fraction facing an aggregate shock from the expression.

Instead, I analyze the variation of entrepreneurs' income over the cycle numerically by considering three cases: when the economy is in the steady state and when the economy suffers a negative and a positive shock. Figure 1 plots the cumulative distribution function for entrepreneurs when one standard deviation of negative or positive aggregate shock hits the economy and when the economy stays in steady state. The low state enlarges the income gap between rich and poor entrepreneurs since idiosyncratic investment risks rise in the recession. The increased individual difference magnifies the damage of low state to the poor, but reduces it to the rich. The expansion witnesses a similar change but with an opposite direction and a smaller scale. Thus the possibility of obtaining lower income in the expansion is much less than that of higher income in the recession. To sum up, the income share of rich entrepreneurs exhibits countercyclicality while that of poor entrepreneurs shows procyclicality.

When I add up the income distribution of workers and entrepreneurs, the bottom income group consists of poor workers and poor entrepreneurs. Thus, income share of poor agents must show procyclicality. Entrepreneurs account for a large number of top earners, so rich agents in my model see countercyclical income share. The sign of correlations of income shares in the middle groups is indeterminate, but less countercyclical than the top end and less procyclical than the bottom end. It also explains why cyclicality of income shares generally decreases with the income.

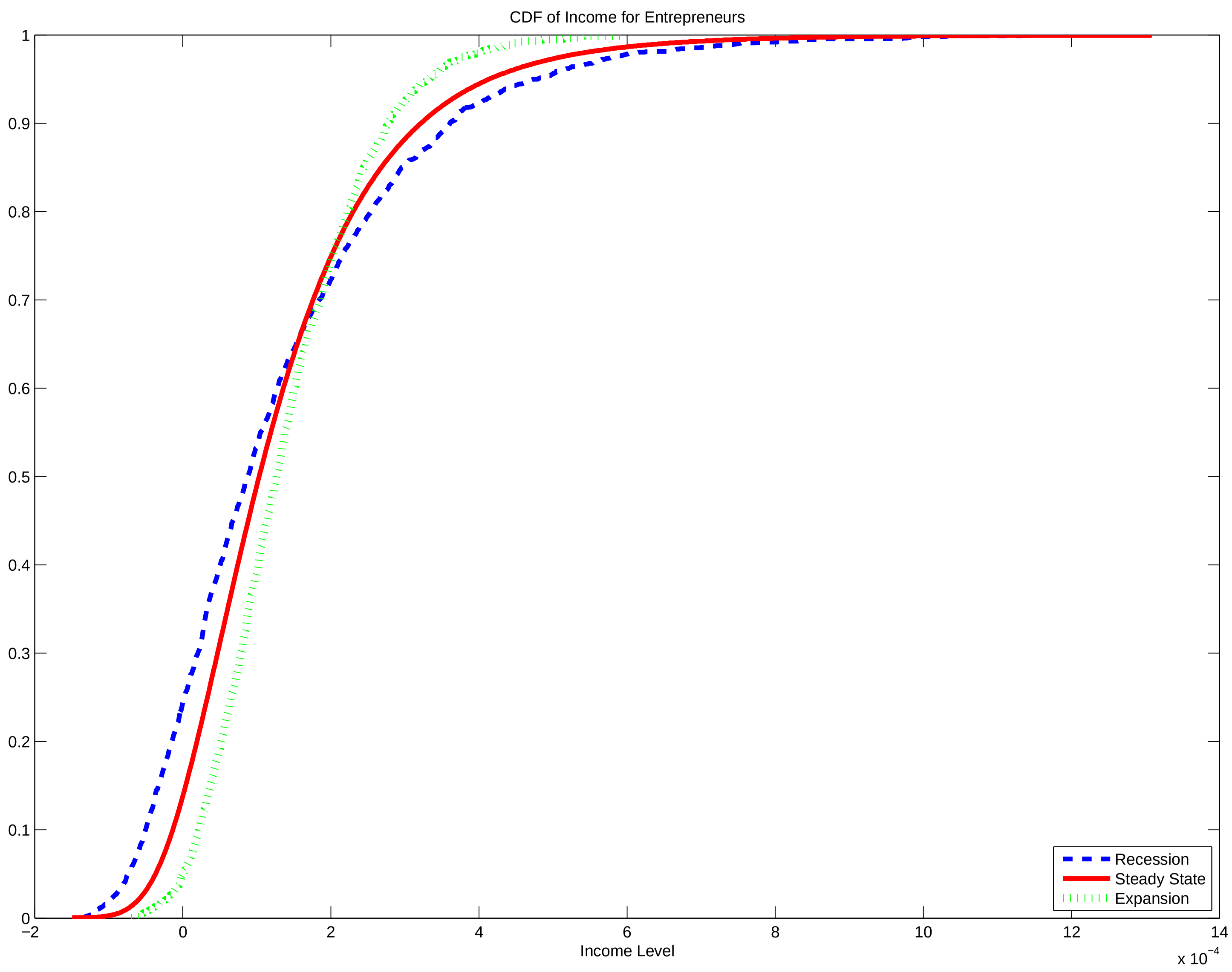


Figure 1: CDF of Entrepreneurs in Recession, Normal Time and Expansion

5 Fiscal Policy Experiments

This section explores if heterogeneity impacts the choice of fiscal policies from a normative perspective. The government, fixing the steady state level of fiscal policies, adjusts the cyclicality of policy instruments to maximize the welfare of different agents. I put emphasis on the time to tax capital more and the time to issue more debt. I find a welfare conflict between entrepreneurs and workers. under the baseline calibration, the government should tax capital more and issue less debt in the recession. However, the welfare conflict implies that the government may cut the capital income tax rate in the recession if it gives more welfare weight to workers, which overturns the finding of Chari, Christiano and Kehoe (1994).

5.1 Objective and Method of Policy Experiments

As the welfare criterion, I adopt the ex-ante utilitarian social utility in which the social planner sums up the life-time individual utility across agents,

$$U_0^{SP} = \int_i \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_s^t \frac{(c_t^i)^{1-\gamma}}{1-\gamma} + \int_j \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_s^t \frac{(c_t^j)^{1-\gamma}}{1-\gamma} = \int_i V(x_0^i) + \int_j V(x_0^j).$$

I denote the value function of entrepreneurs and workers as $\int_i V(x_0^i)$ and $\int_j V(x_0^j)$, respectively. Recall that the value function of entrepreneurs is showed as (7),

$$V(x_0^i) = \frac{\nu_0^{-\gamma} x_{i,0}^{1-\gamma}}{1-\gamma}.$$

Given that the initial conditions of capital and bond holdings are identical under different policy comparison, the value function of entrepreneurs maps one-to-one to the marginal propensity to consume. Moreover, since the risk aversion degree γ is larger than one in my welfare analysis, the life-time utility for entrepreneurs increases with the marginal propensity to consume. In another word, more consumption conditional on a certain available wealth heightens the welfare of entrepreneurs.

Besides, Appendix D derives the value function of workers and I replicate the result:

$$V(x_0^j) = \lambda^{\gamma} \sum_{t=0}^{\infty} \beta_w^t \mathbb{E}_0 \left\{ \frac{[(1-\tau_t^n) w_t]^{1-\gamma}}{1-\gamma} \right\} \exp\left(\frac{\sigma_w^2 (1-\gamma)^2}{2(1-\rho_w^2)} \right).$$

The government searches for the time to tax capital more by changing the responses of tax rate and of debt to output, and comparing the social utility. The economy starts from the steady state. I do not compute the effect of a transition from a previous policy to a new one on the welfare in this subsection, therefore the normative result on the fiscal policies applies to the long-run analysis. I compute the consumption equivalent variation in percentage of the aggregate life-time utility across entrepreneurs, across workers and across agents, as the response of tax rate to output and the response of debt to output range from -2 to 2 and from -1 to 1, respectively, under the baseline calibration of other parameters previously used to produce the dynamics of income distribution. The base to compute the consumption equivalent variation is the life-time utility under the baseline calibration to match the policy data, for each type of agents and the counterpart of social utility. I normalize the value of the base as zero.

5.2 Welfare Conflict between Entrepreneurs and Workers

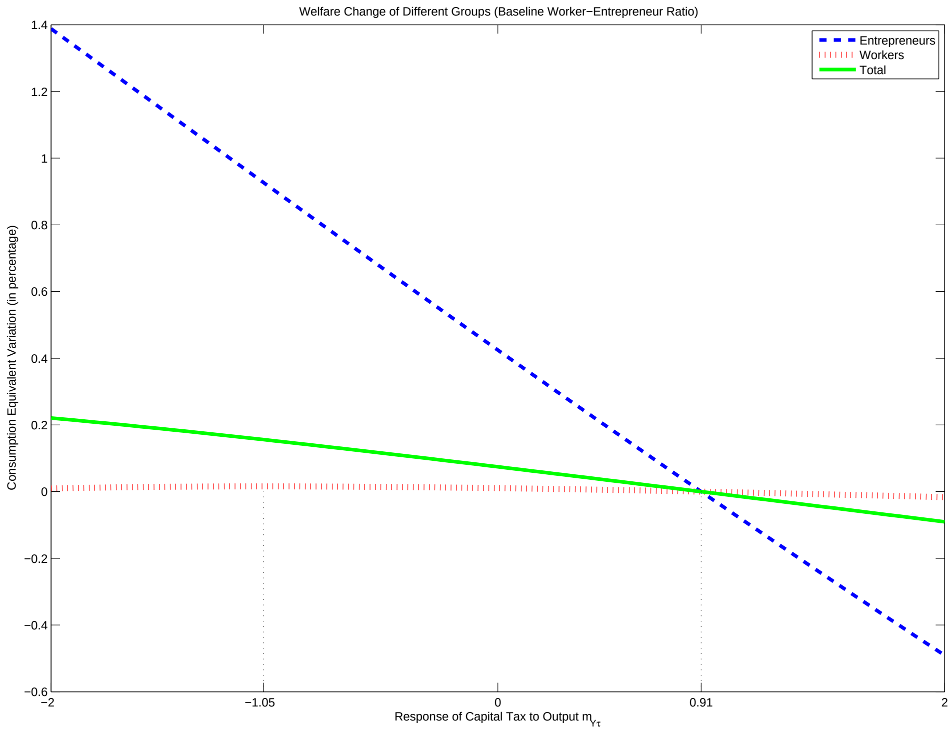
Figure 2(a) shows the welfare change in terms of the consumption equivalent variation of entrepreneurs, workers and all agents under different capital tax policies when the debt policy is fixed. Figure 2 (b), instead, depicts the welfare change of different groups under various debt policies if the government fixes the capital tax policy. They both manifest that entrepreneurs and workers favor different policies. In the recession, entrepreneurs prefer a low debt level and a high capital tax rate while workers, a high debt level and a relatively lower capital tax rate compared to the one favored by entrepreneurs. Figure A1 in Appendix E plots the welfare change if the government simultaneously chooses two fiscal policies. It also indicates a welfare conflict between entrepreneurs and workers under a certain combination of fiscal policies.

To better compare the effect of the cyclicality of fiscal policies, I pick the combinations of the responses of capital tax rate and debt to output which maximize the welfare of entrepreneurs, workers and all agents, respectively. Table 6 reports the welfare change of different agents and the change in aggregate variables under the three policy combinations. All the values are percentage changes compared to the counterparts under the benchmark calibration.

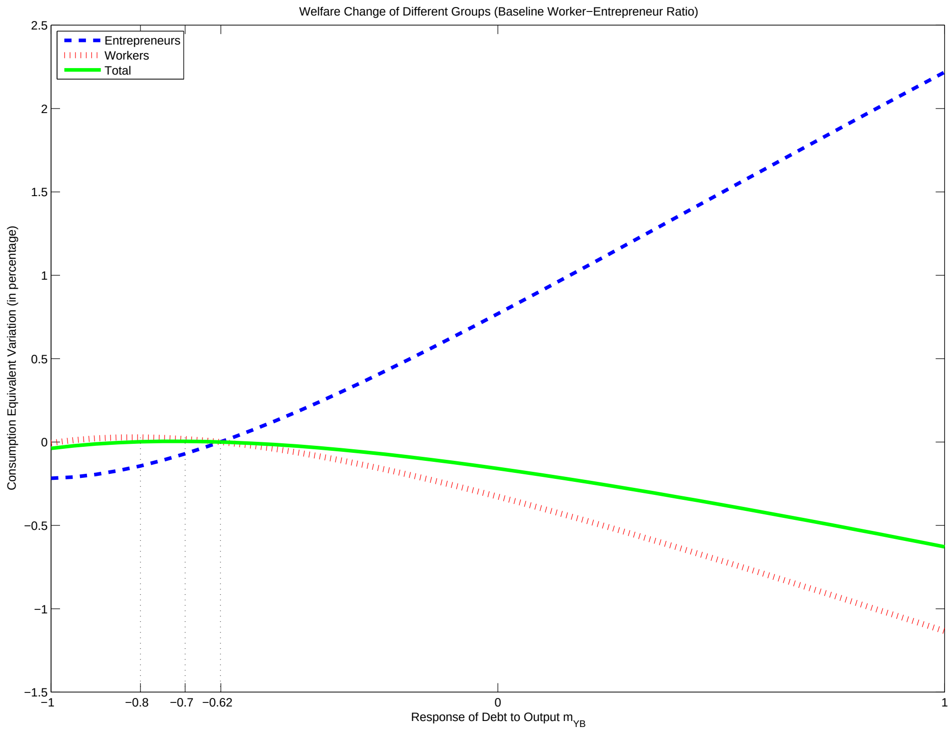
If the government aims to maximize the welfare of entrepreneurs, a high capital tax rate and a low debt level should be applied in the recession. The mean level of bonds rises while the average capital tax rate declines compared to the benchmark calibration. More government bonds give entrepreneurs insurance and crowd out capital, yet less tax burden encourages entrepreneurs to invest more in capital. The combination of policies turns out to slightly enhance capital stock, which furthermore heightens the wage. The mean consumption of entrepreneurs increases due to less tax burden and higher asset income. The government budget balance requires a higher average labor tax rate, which causes a decrease in consumption of workers.

The volatility of consumption of entrepreneurs increases by more than 100% compared to the baseline policy combination. Yet the standard deviation of consumption of entrepreneurs still accounts for less than 5 % of the mean. For entrepreneurs, the increase in the mean consumption outweighs the welfare loss from a higher volatility. Workers see less mean consumption and a higher volatility. Thus, entrepreneurs are better off and workers are worse off.

The policy combination which maximizes the welfare of workers features the debt and the capital tax rate negatively correlated with output. The response of the capital tax rate to output, m_{Y_T}, is higher than the policy prescription to optimize the welfare of entrepreneurs. Under this set of policies, the mean levels of bonds and the capital tax rate decrease. The crowd-in effect leads to higher capital stock and a higher wage. The consumption of entrepreneurs increases, but less than under the policies maximizing entrepreneurs' welfare. The current policies also result in higher consumption of workers even though the labor tax rate increases. Both the consumption of entrepreneurs and workers varies more than under the baseline calibration, but less violently than under the previous policy combination. Therefore, both types of agents are better off.



(a) Welfare Change as $m_{Y\tau}$ Changes (Baseline Worker-Entrepreneur Ratio)



(b) Welfare Change as m_{YB} Changes (Baseline Worker-Entrepreneur Ratio)

Figure 2: Welfare Change of Different Groups

5.3 Time to Tax Capital and Increase Debt

Chari, Christiano and Kehoe (1994) investigate optimal fiscal policies in the business cycle. They find that the correlation of an optimal capital income tax rate with technology shock is negative with uncontingent debt in their baseline model. In addition, when there is a negative innovation to the technology shock or a positive innovation to government consumption, there is a positive innovation in the tax on private assets. They argue that it is efficient for these shocks which affect the government budget constraint to be absorbed mainly by the tax on private assets.

The policy that maximizes the social utility in my model features a 0.45 percent increase in the capital tax rate and a 0.59 percent decrease in the debt level when the output drops by one percent. The policy combination results in a low average level in debt and a low average capital tax rate. Capital stock and output increase by 1.03% and 0.37%, respectively, compared to the baseline policies, followed by a higher wage. The policies also lead to a higher labor tax rate. The entrepreneurs have more average consumption while workers have slightly less consumption. The volatility of the consumption of entrepreneurs and workers see opposite changes; in particular, the consumption of entrepreneurs fluctuates by a much larger margin. The mean effect dominates the fluctuation effect for entrepreneurs. The large decrease in the volatility of consumption of workers outweighs the reduced mean consumption. Therefore, both entrepreneurs and workers are better off; but entrepreneurs benefit more.

The result seems to confirm the finding of Chari, Christiano and Kehoe (1994). Yet the capital rate under the policy combination maximazing social welfare increases in the recession less than under the one favoring only entrepreneurs. It implies that considering workers' welfare lowers the capital tax that should be levied in the bust. The utilitarian social utility weighs less the equality compared to many other welfare criteria. The government would specify a capital tax rule in which the rate increases little or even decreases in the recession if the government puts sufficient emphasis on workers, or poor agents on average. Therefore, it is possible that the government should cut the capital tax rate in the recession.

5.4 Impulse Responses of Aggregate Variables under Different Policies

I plot the impulse responses of aggregate variables to a standard deviation of negative aggregate productivity shock under the baseline calibration and the other three policy combinations optimizing entrepreneurs, workers and all agents, respectively.

The baseline policy combination shows a different pattern in the dynamics of the capital tax rate after a negative productivity shock: the government cuts the tax rate as specified in the rule. The policy which maximizes the welfare of entrepreneurs requires a low debt level while other policies indicate higher debt than in the normal time. The difference in the change of debt causes the qualitative disparity in the change of portfolio choice. All four policies discussed lead to lower marginal propensity to consume, or equivalent to say, entrepreneurs save more in the recession. Only under the policy maximizing entrepreneurs' welfare do investors hold more capital. The risk premium increases, although the average rental rate of capital and interest rate both decline, because investors tend to hold safe assets after confronting much higher idiosyncratic investment risks in the recession. Generally speaking, the higher capital tax rate and the less

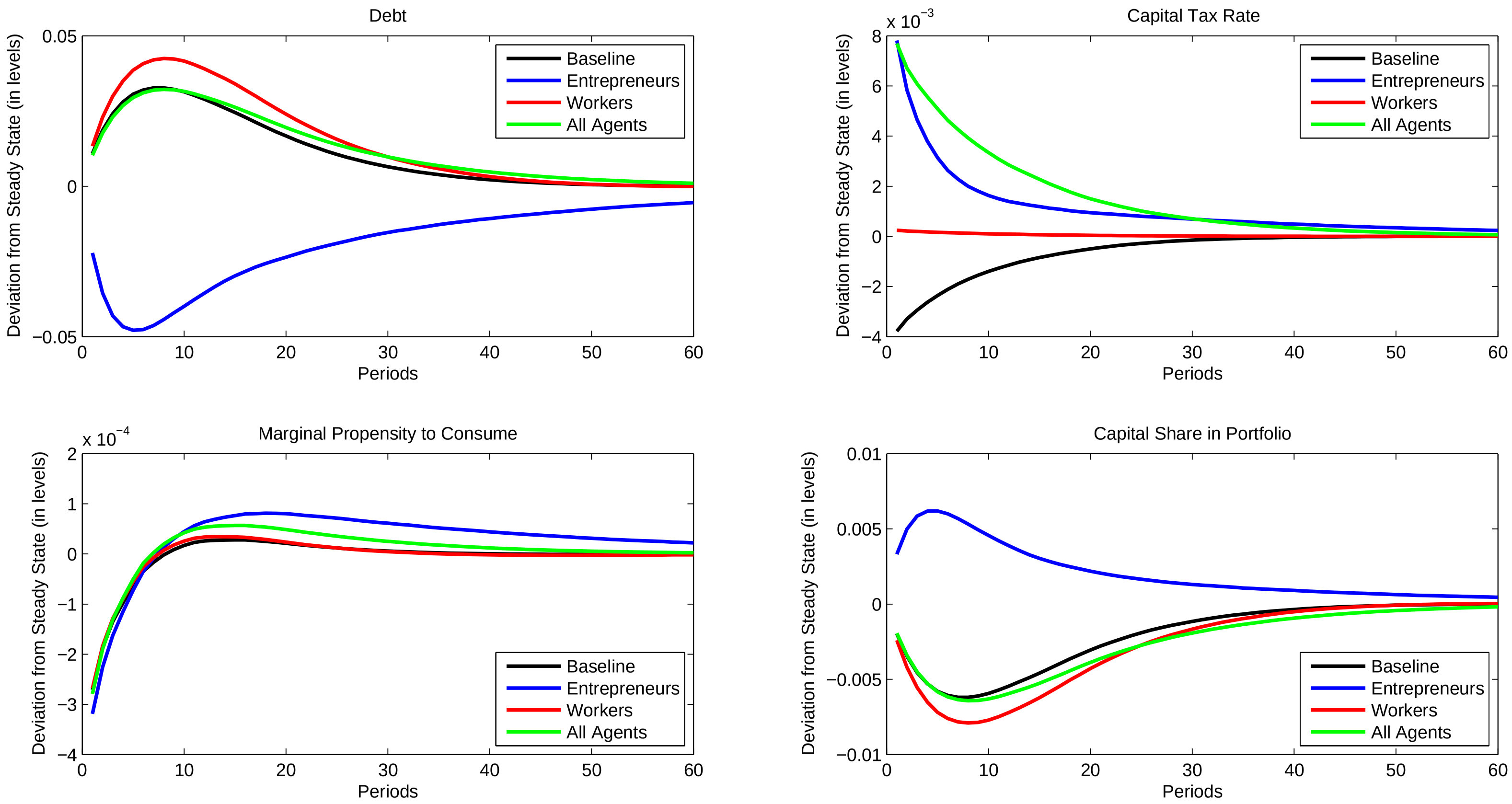


Figure 3: Impulse Responses under Different Policies

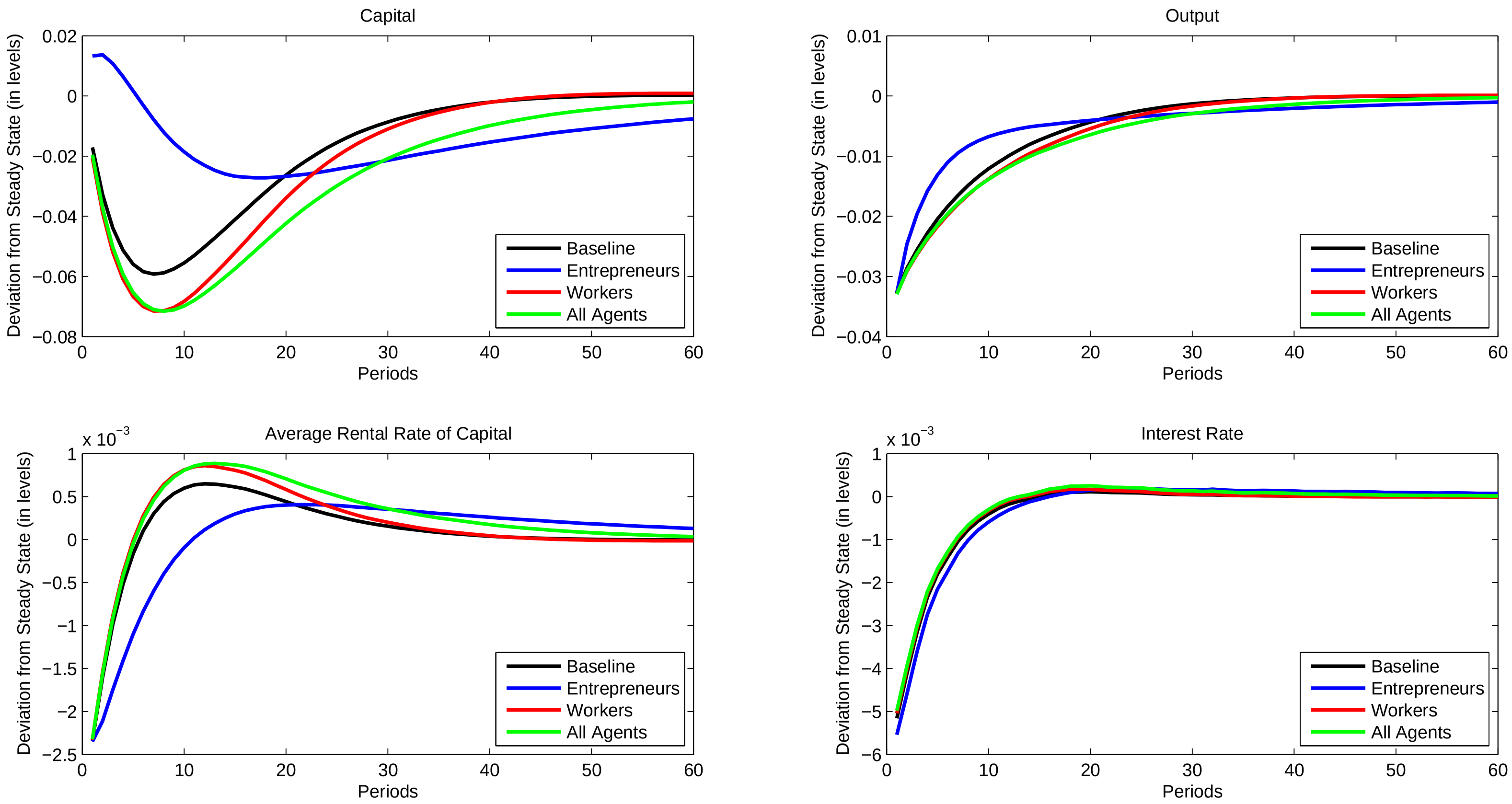


Figure 3: Impulse Responses under Different Policies (Continued)

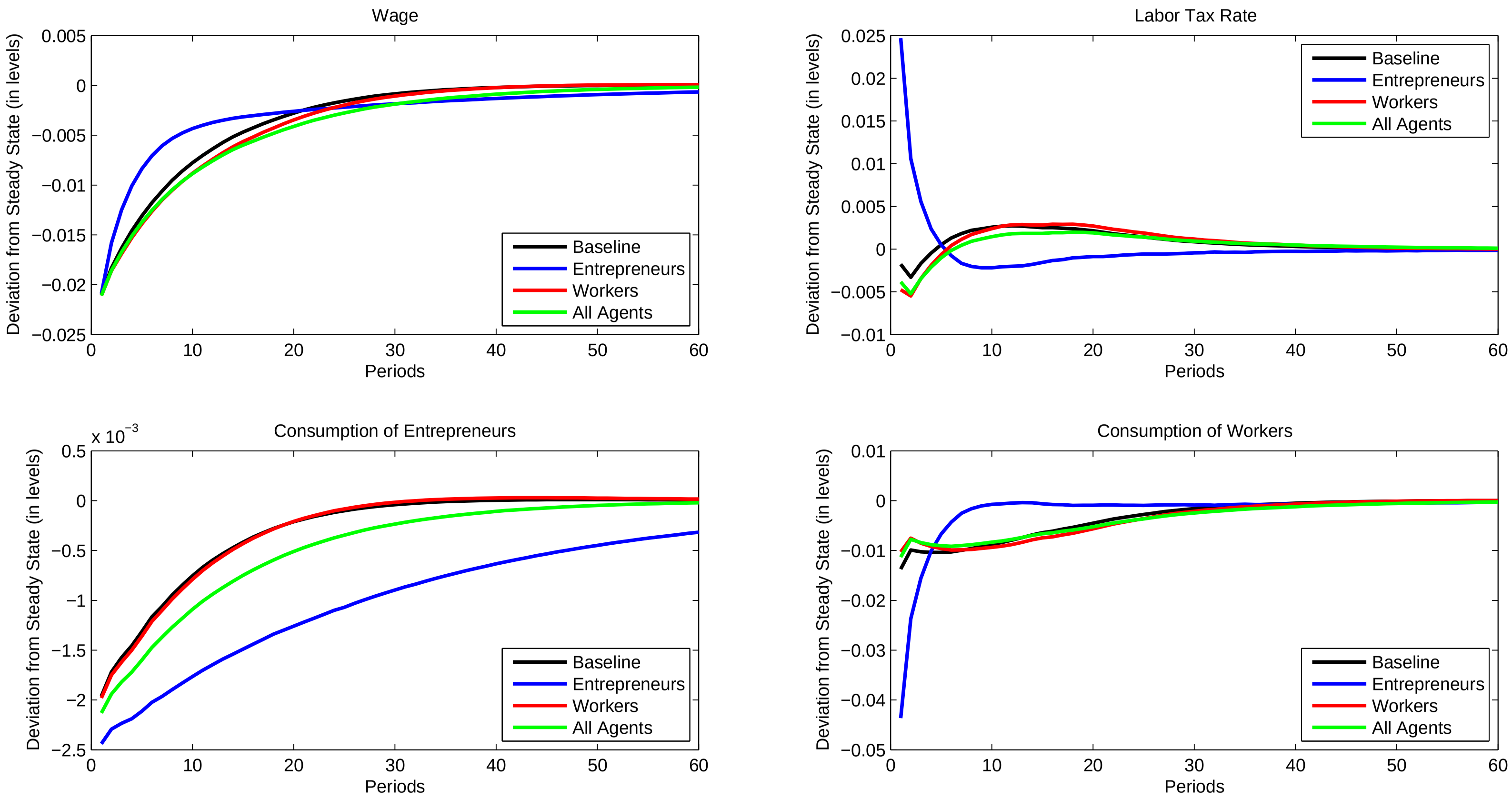


Figure 3: Impulse Responses under Different Policies (Continued)

debt when the adverse shock hits, the more capital and output in the early stage after the shock. However, the recovery of capital slows down and falls behind the counterparts under other policy combinations because the average return to capital is lower. Entrepreneurs see a lower consumption under each policy combination; among all them, entrepreneur-preferred policies pull down the consumption most and cause the slowest recovery.

The low output and the low wage result from low productivity. The labor tax rate rises merely under the entrepreneur-preferred policy combination, which lowers the consumption of workers more than other policies. Yet since the policy maximizing entrepreneurs' welfare also raises the capital stock in the first several periods after the shock, the output and the wage recover faster and so does the consumption of workers.

5.5 Constant Labor Tax Rate

The above subsections indicate that the change of cyclical behavior of fiscal policies redistributes the allocations between entrepreneurs and workers in the long run. Consequently, the welfare of entrepreneurs and workers witnesses different reactions. Since I use labor tax rate to balance the government budget, if the government raises debt or lowers capital tax rate, labor tax rate may have to rise. This subsection conducts an experiment in which the government adjusts the debt according to the policy specification (9) while keeping the same level of labor tax rate but applying capital tax rate to balance the government budget. It directly affects entrepreneurs' welfare through saving and portfolio choices and indirectly impacts workers due to resulting wages.

I find that the welfare conflict still occurs in this experiment. The patterns of the welfare change of three groups are similar to what is shown in Panel (b) of Figure 2 with the time-varying labor tax rate. Entrepreneurs prefer a low debt level in the recession while workers prefer a high level. Compared to the case of the time-varying labor tax rate, the policy that maximizes workers' welfare demands a lower volatility of debt. The utilitarian social utility requires a debt policy close to the one maximizing workers, with an even lower volatility.

6 Robustness Check

This section examines the robustness of the result of the welfare conflict. First, I let the volatility of idiosyncratic investment risks uncorrelated to the aggregate productivity, $\sigma_{e,t}^2 = \sigma_e^2$ and simulate the welfare of heterogeneous agents under different policies. Second, I shut down the idiosyncratic investment risks, $\sigma_{e,t} = 0$, to see its effect on the welfare. The welfare conflict exists with constant volatility of idiosyncratic investment risks, but disappears under reasonable capital tax policies without these risks.

6.1 Constant Volatility of Idiosyncratic Investment Risks

This subsection carries out the above policy experiment in which the government chooses both the responses of fiscal policy instruments to output except that the volatility of idiosyncratic investment risks is set to be constant, $\sigma_{e,t}^2 = \sigma_e^2$. Figure 5 plots the welfare change of entrepreneurs, workers and

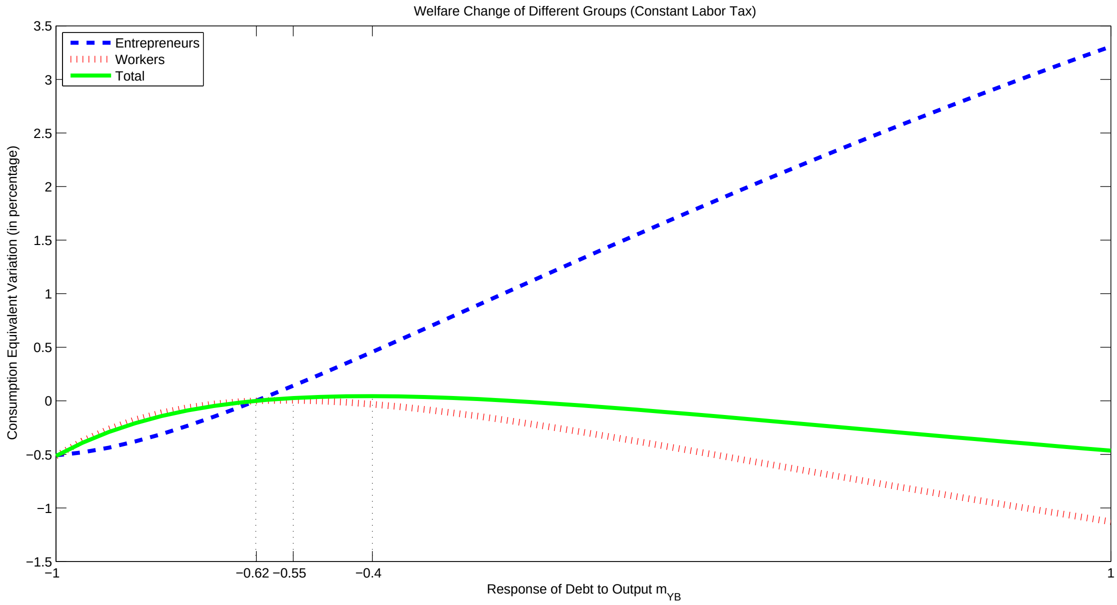


Figure 4: Welfare Change as m_{YB} changes under Constant Labor Tax Rate

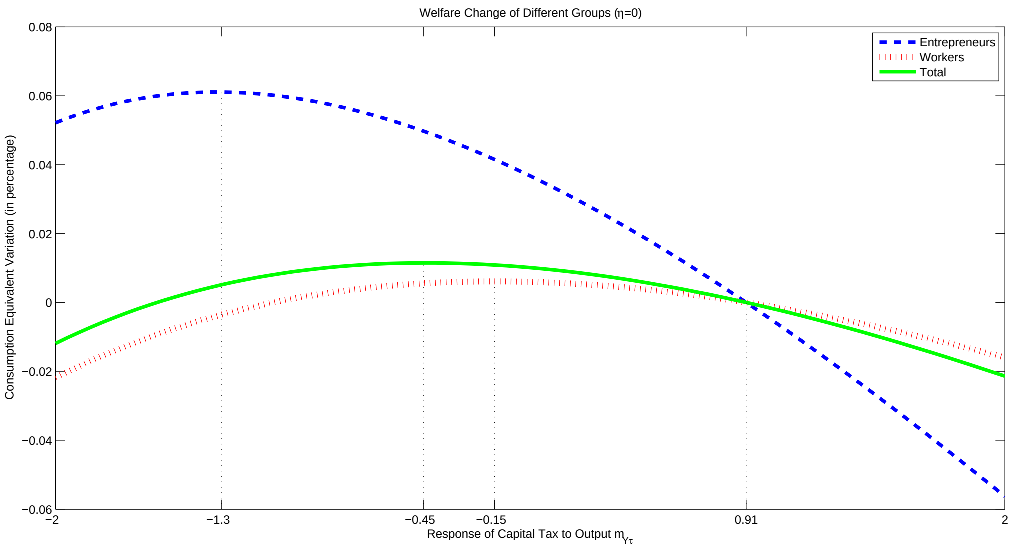
social utility, if the government only modifies the response of the capital tax rate to output, $m_{Y\tau}$, or if the government only changes the response of the debt to output, m_{YB}.

A comparison of Figure 2 and 5 clearly shows a difference in the welfare change with countercyclical and constant idiosyncratic investment risks. First, countercyclical idiosyncratic investment risks enlarge the scale of welfare change. Second, countercyclical risks influence the choice of fiscal policies which aim to maximize the welfare of certain groups. With constant idiosyncratic investment risks, entrepreneurs prefer a higher tax rate in the recession, but much lower than the preferred policy with countercyclical idiosyncratic risks. The policy maximizing workers' welfare specifies the capital tax rate to be close to constant over the cycle. If the rule of the capital tax rate is fixed, entrepreneurs turn to a higher debt level in the recession, or in general, a volatile debt policy, while workers prefer a relatively constant debt policy. In a nutshell, countercyclical idiosyncratic investment risks magnifies the fluctuations of capital and output so that investors are inclined to the debt policy which helps to smooth the investment over the cycle, and the capital tax rate which encourages more investment in the expansion. The result of the welfare conflict between entrepreneurs and workers still holds with constant idiosyncratic investment risks even though the current risks narrow the disparity of preferred policies by different groups.

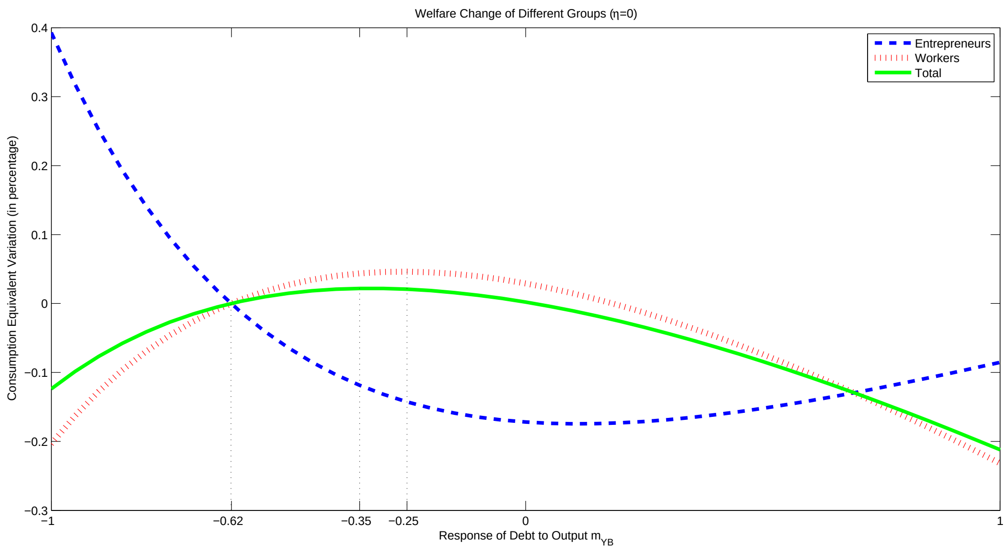
6.2 No Idiosyncratic Investment Risks

This subsection shuts down idiosyncratic investment risks and reassesses fiscal policies. Notice that the model features heterogeneity even without idiosyncratic risks because entrepreneurs and workers are assumed to receive income from different sources. I plot the welfare change of entrepreneurs, workers and social utility under different capital tax policies if the debt policy is fixed, or under different debt policies if the capital tax policy is fixed.

Figure 6 shows a similar pattern in the welfare change of the three groups with two exceptions: first,

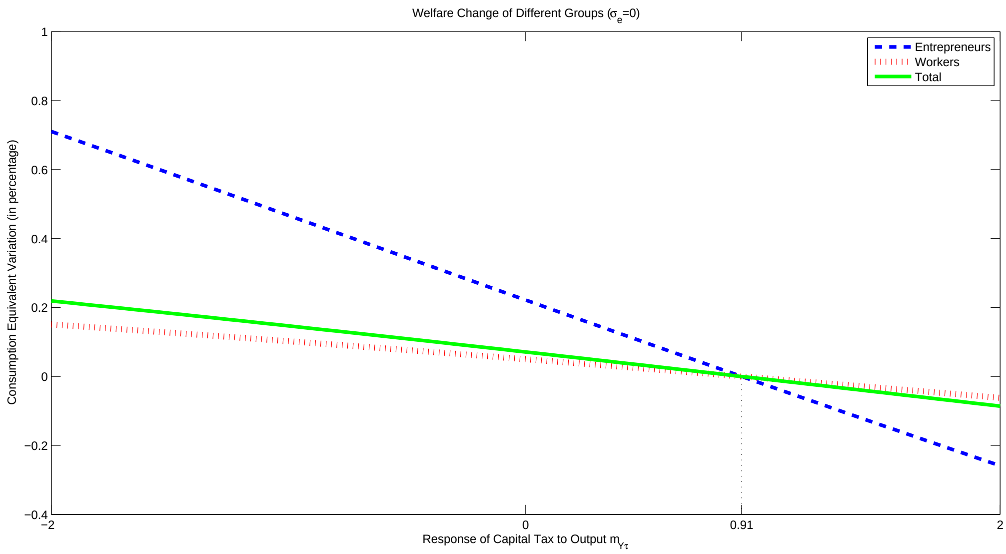


(a) Welfare Change as $m_{Y\tau}$ Changes (Constant Idiosyncratic investment risks)

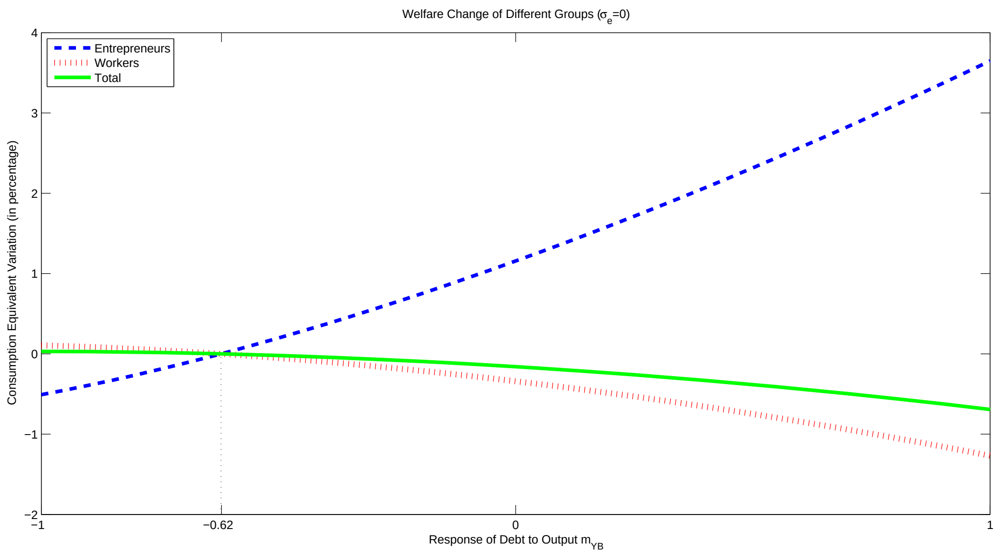


(b) Welfare Change as m_{YB} Changes (Constant Idiosyncratic investment risks)

Figure 5: Welfare Change of Different Groups with Constant Idiosyncratic investment risks



(a) Welfare Change as $m_{Y\tau}$ Changes (No Idiosyncratic investment risks)



(b) Welfare Change as m_{YB} Changes (No Idiosyncratic investment risks)

Figure 6: Welfare Change of Different Groups without Idiosyncratic investment risks

the welfare change varies in a smaller range; second, the tax policy seems able to improve both groups. However, since the welfare change of different groups differs in the declining margins, it is unclear that the welfare conflict fails to appear without idiosyncratic investment risks since the government may tax capital sufficiently much in the recession. I modify the response parameter value downward to $m_{Y\tau} = -12$, which points to an unrealistically volatile tax policy, yet still no conflict has been spotted. Therefore, at least no reasonable capital tax policy can generate the welfare conflict.

The welfare conflict under the debt policy is similar to the baseline policy experiment. Entrepreneurs and workers prefer different changes in debt levels over the business cycle.

To sum up, idiosyncratic investment risks matter for the welfare conflict between entrepreneurs and workers under reasonable capital tax policies, but not under debt policies.

7 Conclusion

This paper focuses on studying the effect of fiscal policies over the business cycle on investment and the welfare of heterogeneous agents who differ in idiosyncratic production and labor income risks. I use the framework to first generate the level and dynamics of income distribution and then conduct several fiscal policy experiments.

My first contribution is to extend the framework of Angeletos (2007) to aggregate shocks. Entrepreneurs in my model are exposed to not only idiosyncratic investment risks, but also aggregate shocks. Secondly, I achieve to match the level and dynamics of income inequality as in the US data.

I answer the question of when to tax capital income and when to issue more debt, both of which are rarely explored in the literature. The policy experiment emphasizes a welfare conflict between entrepreneurs and workers. The result is robust to constant volatility of idiosyncratic investment risks. But the welfare conflict disappears after removing idiosyncratic investment risks.

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Appendix A

Proof of Lemma 1

I start from showing the characterization of allocations and choices in the individual level.

The Euler equations derived from the entrepreneur optimization problem are

$$(c_t^i)^{-\gamma} = \beta_s \mathbb{E}_t\left[(c_{t+1}^i)^{-\gamma}(1-\tau_{t+1}^a)(r(A_{t+1}^i, w_{t+1}) + 1 - \delta)\right], \tag{25}$$

$$(c_t^i)^{-\gamma} = \beta_s \mathbb{E}_t\left[(c_{t+1}^i)^{-\gamma}(1-\tau_{t+1}^a)R_{t+1}\right]. \tag{26}$$

I guess that the solution to the entrepreneur optimization problem is as follows:

$c_t^i = (1-\nu_t)x_t^i$, and $k_{t+1}^i = \nu_t\phi_t x_t^i$, so $b_{t+1}^i = \nu_t(1-\phi_t)x_t^i$ from the budget constraint of the entrepreneur. I will determine later the coefficients ν_t and ϕ_t which only depend on the current aggregate state. To simplify the notation, I define the aggregate state at t, $s_t = (K_t, B_t, z_t, g_t)$. Then I write $\nu_t = \nu_t(s_t)$ and $\phi_t = \phi_t(s_t)$. With some algebra,

$$\begin{aligned} x_{t+1}^i &= (1-\tau_{t+1}^a)[(r(A_{t+1}^i, w_{t+1}) + 1 - \delta)k_{t+1}^i + R_{t+1}b_{t+1}^i] \\ &= (1-\tau_{t+1}^a)[\phi_t(s_t)(r(A_{t+1}^i, w_{t+1}) + 1 - \delta) + (1-\phi_t(s_t))R_{t+1}]\nu_t(s_t)x_t^i. \end{aligned} \tag{27}$$

Then the first Euler equation becomes

$$\begin{aligned} (1-\nu_t(s_t))^{-\gamma}(x_t^i)^{-\gamma} &= \beta_s \mathbb{E}_t\{[(1-\nu_{t+1}(s_{t+1}))x_{t+1}^i]^{-\gamma}(1-\tau_{t+1}^a)(r(A_{t+1}^i, w_{t+1}) + 1 - \delta)\} \\ &= \beta_s \nu_t(s_t)^{-\gamma}(x_t^i)^{-\gamma}\mathbb{E}_t\left\{(1-\nu_{t+1}(s_{t+1}))^{-\gamma}(1-\tau_{t+1}^a)^{1-\gamma}\right. \\ &\quad \left.[\phi_t(s_t)(r(A_{t+1}^i, w_{t+1}) + 1 - \delta) + (1-\phi_t(s_t))R_{t+1}]^{-\gamma}(r(A_{t+1}^i, w_{t+1}) + 1 - \delta)\right\}. \end{aligned}$$

Then we cross out the same factors from both handsides,

$$\begin{aligned} (1-\nu_t(s_t))^{-\gamma} &= \beta_s \nu_t(s_t)^{-\gamma}\mathbb{E}_t\{(1-\nu_{t+1}(s_{t+1}))^{-\gamma}(1-\tau_{t+1}^a)^{1-\gamma} \\ &\quad [\phi_t(s_t)(r(A_{t+1}^i, w_{t+1}) + 1 - \delta) + (1-\phi_t(s_t))R_{t+1}]^{-\gamma}(r(A_{t+1}^i, w_{t+1}) + 1 - \delta)\}. \end{aligned} \tag{28}$$

Likewise, the second Euler equation can be transformed as

$$\begin{aligned} (1-\nu_t(s_t))^{-\gamma} &= \beta_s \nu_t(s_t)^{-\gamma}\mathbb{E}_t\{(1-\nu_{t+1}(s_{t+1}))^{-\gamma}(1-\tau_{t+1}^a)^{1-\gamma} \\ &\quad [\phi_t(s_t)(r(A_{t+1}^i, w_{t+1}) + 1 + \delta) + (1-\phi_t(s_t))R_{t+1}]^{-\gamma}R_{t+1}\}. \end{aligned} \tag{29}$$

Combining these two equations, we obtain the equality between the gross returns of risky and risk-free assets:

$$
\begin{aligned}
0 = & \mathbb{E}_t\{(1-\nu_{t+1}(s_{t+1}))^{-\gamma}(1-\tau^a_{t+1})^{1-\gamma}[\phi_t(s_t)(r(A^i_{t+1}, w_{t+1}) + 1 - \delta) + \\
& + (1-\phi_t(s_t))R_{t+1}]^{-\gamma}(r(A^i_{t+1}, w_{t+1}) + 1 - \delta - R_{t+1})\} \\
= & \int_0^\infty \left(\int_0^\infty (1-\nu_{t+1}(s_{t+1}))^{-\gamma}(1-\tau^a_{t+1})^{1-\gamma}[\phi_t(s_t)(r(e^i_{t+1}, \epsilon^z_{t+1}, w(\epsilon^z_{t+1}); z_t) + 1 - \delta) + \right. \\
& \left. +(1-\phi_t(s_t))R_{t+1}]^{-\gamma}(r(e^i_{t+1}, \epsilon^z_{t+1}, w(\epsilon^z_{t+1}); z_t) + 1 - \delta - R_{t+1}) \, \mathrm{d}F(e^i_{t+1}) \right) \, \mathrm{d}F(\epsilon^z_{t+1}).
\end{aligned}
\tag{30}
$$

Multiply the above Equations 26 and 27 with ϕ_t and $1-\phi_t$, respectively, and sum up to get:

$$
\begin{aligned}
(1-\nu_t(s_t))^{-\gamma} = \beta_s \nu_t(s_t)^{-\gamma} & \mathbb{E}_t\{(1-\nu_{t+1}(s_{t+1}))^{-\gamma}(1-\tau^a_{t+1})^{1-\gamma} \\
& [\phi_t(s_t)(r(A^i_{t+1}, w_{t+1}) + 1 - \delta) + (1-\phi_t(s_t))R_{t+1}]^{1-\gamma}\}.
\end{aligned}
\tag{31}
$$

Define $\phi_t = \arg\max_{\phi \in [0,1]} \mathbb{CE}_t \left\{ \phi_t(r(A^i_{t+1}, w_{t+1}) + 1 - \delta) + (1-\phi_t)R_{t+1} \right\}$. In particular,

$$
\begin{aligned}
& \mathbb{CE}_t \left\{ \phi_t(r(A^i_{t+1}, w_{t+1}) + 1 - \delta) + (1-\phi_t)R_{t+1} \right\} \\
= & \left[\int_{-\infty}^\infty \left(\int_{-\infty}^\infty [\phi_t(s_t)(r(e^i_{t+1}, \epsilon^z_{t+1}, w(\epsilon^z_{t+1}); z_t) + 1 - \delta) + (1-\phi_t(s_t))R_{t+1}]^{1-\gamma} \mathrm{d}F(e^i_{t+1}) \right) \mathrm{d}F(\epsilon^z_{t+1}) \right]^{\dfrac{1}{1-\gamma}}.
\end{aligned}
\tag{32}
$$

The process requires numerical solutions. The preceding integral demonstrates that the optimal saving and portfolio choices depend only on the aggregate state. I drop the notation of aggregate state since now.

Given ϕ_t and τ^a_{t+1}, ν_t can be computed by (38).

Next I derive the functional form of the value function for entrepreneurs. I first guess that the value function $V(x^i_t) = \psi_t u(x^i_t)$. From the envelope theorem, $V'(x^i_t) = u'(c^i_t)$, which is

$$
\psi_t (x^i_t)^{-\gamma} = (c^i_t)^{-\gamma} = [(1-\nu_t)(x^i_t)]^{-\gamma}
$$

I simplify the above equation by crossing out the common factor so that

$$
\psi_t = (1-\nu_t)^{-\gamma}. \tag{33}
$$

Hence, $V(x^i_t) = \dfrac{(1-\nu_t)^{-\gamma}(x^i_t)^{1-\gamma}}{1-\gamma}$.

Now I verify whether the value function obtained fits the entrepreneur's optimization problem. Notice that $V(x^i_t) = \dfrac{(c^i_t)^{1-\gamma}}{1-\gamma} + \beta_s \mathbb{E}_t V(x^i_{t+1})$. I insert the expressions of the value function and consumption, Eq. (13), into the equation.

$$
\frac{(1-\nu_t)^{-\gamma}(x^i_t)^{1-\gamma}}{1-\gamma} = \frac{[(1-\nu_t)(x^i_t)]^{1-\gamma}}{1-\gamma} + \beta_s \mathbb{E}_t \left[\frac{(1-\nu_{t+1})^{-\gamma} x^{1-\gamma}_{i,t+1}}{1-\gamma} \right]
$$

I multiply both handsides by $(1-\gamma)$, remove the first term on the right handside to the left, and combine the two terms on the left handside,

$$\begin{aligned}
&\nu_t(1-\nu_t)^{-\gamma}(x_t^i)^{1-\gamma} \\
&= \beta_s \mathbb{E}_t\left[(1-\nu_{t+1})^{-\gamma} x_{i,t+1}^{1-\gamma}\right] \\
&= \beta_s \mathbb{E}_t\left[(1-\nu_{t+1})^{-\gamma}\left\{(1-\tau_{t+1}^a)[\phi_t(r(A_{t+1}^i, w_{t+1}) + 1 - \delta) + \right.\right. \\
&+ \left.\left. (1-\phi_t)R_{t+1}]\nu_t(x_t^i)\right\}^{1-\gamma}\right].
\end{aligned}$$

Then I eliminate the common factors ν_t and $(x_t^i)^{1-\gamma}$ from both handsides,

$$(1-\nu_t)^{-\gamma} = \beta_s \nu_t^{-\gamma} \mathbb{E}_t\left[(1-\nu_{t+1})^{-\gamma}\left\{(1-\tau_{t+1}^a)[\phi_t(r(A_{t+1}^i, w_{t+1}) + 1 - \delta) + (1-\phi_t)R_{t+1}]\right\}^{1-\gamma}\right],$$

which is validated by (38). □

Appendix B

Effect of idiosyncratic investment risks

This section shows how idiosyncratic investment risks affect the portfolio choice at the steady state. For the exposional convenience, I simply assume the risk aversion degree $\gamma = 1$, that is, the logarithm case.

From the wage determination $w_t = (1-\alpha)(z_t K_t)^{\alpha}$ and the specification of risks, I rewrite the expression of individual capital investment return

$$\begin{aligned} r^i_{t+1} &= \alpha \left(\frac{1-\alpha}{w_{t+1}}\right)^{\frac{1}{\alpha}-1} A^i_{t+1} \\ &= \alpha z^{\alpha-1}_{t+1} K^{\alpha-1}_{t+1} A^i_{t+1} \\ &= \alpha z^{\alpha}_{t+1} K^{\alpha-1}_{t+1} \exp\left[\frac{-\sigma^2_e}{2} + e^i_{t+1}\right]. \end{aligned} \tag{34}$$

I use the second order Taylor series with respect to e^i_{t+1} around 0 to approximate the certainty equivalent.

$$\begin{aligned} &\mathbb{CE}_t\left\{\phi_t(r^i_{t+1}+1-\delta)+(1-\phi_t)R_{t+1}\right\} \\ \approx &\left(\int_{-\infty}^{\infty} [\phi_t(\alpha z^{\alpha}_{t+1}K^{\alpha-1}_{t+1}\exp\left[\frac{-\sigma^2_e}{2}\right]+1-\delta)+(1-\phi_t)R_{t+1}]^{1-\gamma} + \frac{1-\gamma}{2}\sigma^{i2}_e \right. \\ &\left\{-\gamma[\phi_t(\alpha z^{\alpha}_{t+1}K^{\alpha-1}_{t+1}\exp\left[\frac{-\sigma^2_e}{2}\right]+1-\delta)+(1-\phi_t)R_{t+1}]^{-\gamma-1}\left(\phi_t\alpha z^{\alpha}_{t+1}K^{\alpha-1}_{t+1}\exp\left[\frac{-\sigma^2_e}{2}\right]\right)^2 + \right. \\ &\left.\left. +[\phi_t(\alpha z^{\alpha}_{t+1}K^{\alpha-1}_{t+1}\exp\left[\frac{-\sigma^2_e}{2}\right]+1-\delta)+(1-\phi_t)R_{t+1}]^{-\gamma}\phi_t\alpha z^{\alpha}_{t+1}K^{\alpha-1}_{t+1}\exp\left[\frac{-\sigma^2_e}{2}\right]\right\}\mathrm{d}F(\epsilon^z_{t+1})\right)^{\frac{1}{1-\gamma}} \end{aligned} \tag{35}$$

I take the first order condition with respect to ϕ_t to pin down the optimal portfolio choice.

$$\begin{aligned} &\int_{-\infty}^{\infty}\left\{\left[(\alpha z^{\alpha}_{t+1}K^{\alpha-1}_{t+1}\exp\left[\frac{-\sigma^2_e}{2}\right]+1-\delta-R_{t+1})^3+\frac{1-\gamma}{2}\sigma^2_e[(\alpha z^{\alpha}_{t+1}K^{\alpha-1}_{t+1}\exp\left[\frac{-\sigma^2_e}{2}\right])^2\right.\right. \\ &(1-\gamma)+(1-\delta-R_{t+1})\alpha z^{\alpha}_{t+1}K^{\alpha-1}_{t+1}\exp\left[\frac{-\sigma^2_e}{2}\right]](\alpha z^{\alpha}_{t+1}K^{\alpha-1}_{t+1}\exp\left[\frac{-\sigma^2_e}{2}\right]+ \\ &\left.+1-\delta-R_{t+1})\right]\phi^2_t+\left\{2(\alpha z^{\alpha}_{t+1}K^{\alpha-1}_{t+1}\exp\left[\frac{-\sigma^2_e}{2}\right]+1-\delta-R_{t+1})^2+\right. \\ &+\sigma^2_e\left[(1-\gamma)(\alpha z^{\alpha}_{t+1}K^{\alpha-1}_{t+1}\exp\left[\frac{-\sigma^2_e}{2}\right])^2+\alpha z^{\alpha}_{t+1}K^{\alpha-1}_{t+1}\exp\left[\frac{-\sigma^2_e}{2}\right](1-\delta-R_{t+1})\right]- \\ &\left.-\frac{\gamma}{2}\sigma^2_e\alpha z^{\alpha}_{t+1}K^{\alpha-1}_{t+1}\exp\left[\frac{-\sigma^2_e}{2}\right](\alpha z^{\alpha}_{t+1}K^{\alpha-1}_{t+1}\exp\left[\frac{-\sigma^2_e}{2}\right]+1-\delta-R_{t+1})\right\}R_{t+1}\phi_t+ \\ &\left.+R^2_{t+1}\left[(1+\frac{\sigma^2_e}{2})\alpha z^{\alpha}_{t+1}K^{\alpha-1}_{t+1}\exp\left[\frac{-\sigma^2_e}{2}\right]+1-\delta-R_{t+1}\right]\right\}\mathrm{d}F(\epsilon^z_{t+1})=0 \end{aligned} \tag{36}$$

The deterministic steady state rules out the aggregate shocks. I list the certainty equivalent and its derivative with respect to ϕ_t at the steady state in the following.

$$
\begin{aligned}
\overline{\mathbb{CE}} \approx \Bigg(& [\bar{\phi}(\alpha \bar{K}^{\alpha-1} \exp\left[\frac{-\sigma_e^2}{2}\right] + 1 - \delta) + (1-\bar{\phi})\bar{R}]^{1-\gamma} + \frac{1-\gamma}{2}\sigma_e^2 \\
& \Bigg\{ -\gamma[\bar{\phi}(\alpha \bar{K}^{\alpha-1} \exp\left[\frac{-\sigma_e^2}{2}\right] + 1 - \delta) + (1-\bar{\phi})\bar{R}]^{-\gamma-1} \left(\bar{\phi}\alpha \bar{K}^{\alpha-1} \exp\left[\frac{-\sigma_e^2}{2}\right]\right)^2 + \\
& + [\phi_t(\alpha \bar{K}^{\alpha-1} \exp\left[\frac{-\sigma_e^2}{2}\right] + 1 - \delta) + (1-\bar{\phi})\bar{R}]^{-\gamma} \bar{\phi}\alpha \bar{K}^{\alpha-1} \exp\left[\frac{-\sigma_e^2}{2}\right] \Bigg\} \Bigg)^{\frac{1}{1-\gamma}}
\end{aligned} \tag{37}
$$

$$
\begin{aligned}
& \left[(\alpha \bar{K}^{\alpha-1} \exp\left[\frac{-\sigma_e^2}{2}\right] + 1 - \delta - \bar{R})^3 + \frac{1-\gamma}{2}\sigma_e^2 [(1-\gamma)(\alpha \bar{K}^{\alpha-1} \exp\left[\frac{-\sigma_e^2}{2}\right])^2 + \right. \\
& \left. + (1 - \delta - \bar{R})\alpha \bar{K}^{\alpha-1} \exp\left[\frac{-\sigma_e^2}{2}\right]](\alpha \bar{K}^{\alpha-1} \exp\left[\frac{-\sigma_e^2}{2}\right] + 1 - \delta - \bar{R}) \right] \bar{\phi}^2 + \\
& + \Bigg\{ 2(\alpha \bar{K}^{\alpha-1} \exp\left[\frac{-\sigma_e^2}{2}\right] + 1 - \delta - \bar{R})^2 + \sigma_e^2 \left[(1-\gamma)(\alpha \bar{K}^{\alpha-1} \exp\left[\frac{-\sigma_e^2}{2}\right])^2 + \right. \\
& \left. + \alpha \bar{K}^{\alpha-1} \exp\left[\frac{-\sigma_e^2}{2}\right](1 - \delta - \bar{R}) \right] - \frac{\gamma}{2}\sigma_e^2 \alpha \bar{K}^{\alpha-1} \exp\left[\frac{-\sigma_e^2}{2}\right](\alpha \bar{K}^{\alpha-1} \\
& \exp\left[\frac{-\sigma_e^2}{2}\right] + 1 - \delta - \bar{R}) \Bigg\} \bar{R}\bar{\phi} + \bar{R}^2 \left[(1 + \frac{\sigma_e^2}{2})\alpha \bar{K}^{\alpha-1} \exp\left[\frac{-\sigma_e^2}{2}\right] + 1 - \delta - \bar{R} \right] = 0
\end{aligned} \tag{38}
$$

Appendix C

Coefficient of Variation of Income

I first report the coefficient of variation of entrepreneurs' income at $t+1$ conditional on t's information Coef.Var.$_t(I^i_{t+1})$.

Conditional expectation of $t+1$'s income of entrepreneurs at t is

$$\begin{aligned}
&\mathbb{E}_t(I^i_{t+1}) \\
&= \mathbb{E}_t\left[r^i_{t+1}k^i_{t+1} + (R_{t+1}-1)b^i_{t+1}\right] \\
&= \mathbb{E}_t\left\{\nu_t x^i_t\left[\phi_t r^i_{t+1} + (1-\phi_t)(R_{t+1}-1)\right]\right\} \\
&= \nu_t x^i_t\left[\phi_t \alpha z^{\alpha\rho_z}\exp\left(\frac{1}{2}\alpha^2\sigma_z^2\right)K_{t+1}^{\alpha-1} + (1-\phi_t)(R_{t+1}-1)\right];
\end{aligned}$$

Conditional variance of $t+1$'s income of entrepreneurs at t is

$$\begin{aligned}
&\mathrm{Var}_t(I^i_{t+1}) \\
&= \mathrm{Var}_t\left[r^i_{t+1}k^i_{t+1} + (R_{t+1}-1)b^i_{t+1}\right] \\
&= \nu_t^2 x_{i,t}^2 \mathrm{Var}_t\left[\phi_t r^i_{t+1} + (1-\phi_t)(R_{t+1}-1)\right] \\
&= \nu_t^2 x_{i,t}^2 \mathrm{Var}_t\left[\phi_t r^i_{t+1}\right] \\
&= \nu_t^2 x_{i,t}^2 \phi_t^2 \alpha^2 z^{2\alpha\rho_z} K_{t+1}^{2(\alpha-1)}\left\{\exp\left[\sigma_e^2\exp(-\eta\rho_z\log z_t) + \frac{1}{2}(2\alpha-\eta\sigma_e^2)^2\sigma_z^2\right] - \exp\left(\alpha^2\sigma_z^2\right)\right\}.
\end{aligned}$$

Thus, the coefficient of variation is computed as the dividend of the standard deviation and the expectation.

$$\text{Coef.Var.}_t(I^i_{t+1}) = \frac{\phi_t\alpha z^{\alpha\rho_z}K_{t+1}^{\alpha-1}\sqrt{\exp\left[\sigma_e^2\exp(-\eta\rho_z\log z_t)+\frac{1}{2}(2\alpha-\eta\sigma_e^2)^2\sigma_z^2\right]-\exp\left(\alpha^2\sigma_z^2\right)}}{\phi_t\alpha z^{\alpha\rho_z}\exp\left(\frac{1}{2}\alpha^2\sigma_z^2\right)K_{t+1}^{\alpha-1}+(1-\phi_t)(R_{t+1}-1)}.$$

Next I derive the coefficient of variation of workers' income at $t+1$ conditional on t's information Coef.Var.$_t(I^j_{t+1})$.

Conditional expectation of $t+1$'s income of workers at t is

$$\begin{aligned}
&\mathbb{E}_t(I^j_{t+1}) \\
&= \mathbb{E}_t\left[w_{t+1}e^j_{t+1}\right] \\
&= \mathbb{E}_t\left[(1-\alpha)z^{\alpha}_{t+1}K^{\alpha}_{t+1}e^j_{t+1}\right] \\
&= (1-\alpha)z_t^{\alpha\rho_z}K^{\alpha}_{t+1}e^{\rho_w}_{j,t}\exp\left[\frac{1}{2}(\alpha^2\sigma_z^2+\sigma_w^2)\right];
\end{aligned}$$

Conditional variance of $t+1$'s income of workers at t is

$$
\begin{aligned}
& \operatorname{Var}_t(I_{t+1}^j) \\
= & \operatorname{Var}_t\left[w_{t+1} e_{t+1}^j\right] \\
= & \operatorname{Var}_t\left[(1-\alpha) z_{t+1}^{\alpha} K_{t+1}^{\alpha} e_{t+1}^j\right] \\
= & (1-\alpha)^2 z_t^{2\alpha\rho_z} K_{t+1}^{2\alpha} e_{j,t}^{2\rho_w}\left[\exp\left(2\alpha^2\sigma_z^2+2\sigma_w^2\right)-\exp\left(\alpha^2\sigma_z^2+\sigma_w^2\right)\right].
\end{aligned}
$$

Then the coefficient of variation is

$$
\text{Coef.Var.}_t(I_{t+1}^j)=\sqrt{\exp\left(\alpha^2\sigma_z^2+\sigma_w^2\right)-1}.
$$

Appendix D

Welfare for Hand-to-mouth Workers

From (4)

$$\log e_{t+1}^{j} = \rho_w \log e_t^{j} + \epsilon_{t+1}^{j},$$

we have that

$$\mathbb{E}_0 \left(\log e_{j,t}\right) = \mathbb{E}_0 \left(\rho_w \log e_{j,t-1}\right) + \mathbb{E}_0 \left(\epsilon_{j,t}\right) = 0;$$

$$\text{Var}_0 \left(\log e_{j,t}\right) = \frac{\sigma_w^2}{1-\rho_w^2};$$

$$\mathbb{E}_0 \left(e_{j,t}\right) = \exp\left(\frac{\sigma_w^2}{2(1-\rho_w^2)}\right), \forall t;$$

and furthermore,

$$\mathbb{E}_0 \left(e_{j,t}^{1-\gamma}\right) = \exp\left(\frac{\sigma_w^2(1-\gamma)^2}{2(1-\rho_w^2)}\right), \forall t.$$

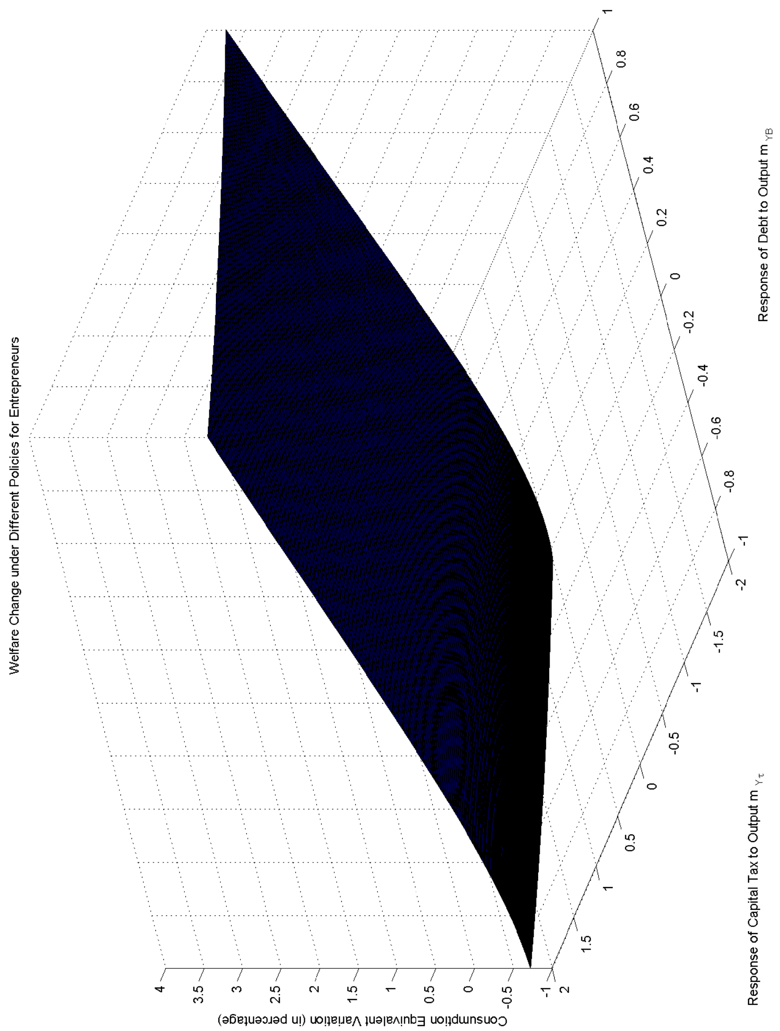
Then the summed unconditional mean of utility at t across workers is

$$\begin{aligned} \int_j \mathbb{E}_0 \left\{\frac{(c_t^j)^{1-\gamma}}{1-\gamma}\right\} &= \int_j \mathbb{E}_0 \left\{\frac{[(1-\tau_t^n)\frac{1}{\lambda}w_t e_{j,t}]^{1-\gamma}}{1-\gamma}\right\} \\ &= \lambda^\gamma \mathbb{E}_0 \left\{\frac{[(1-\tau_t^n)w_t]^{1-\gamma}}{1-\gamma}\right\} \exp\left(\frac{\sigma_w^2(1-\gamma)^2}{2(1-\rho_w^2)}\right). \end{aligned} \tag{39}$$

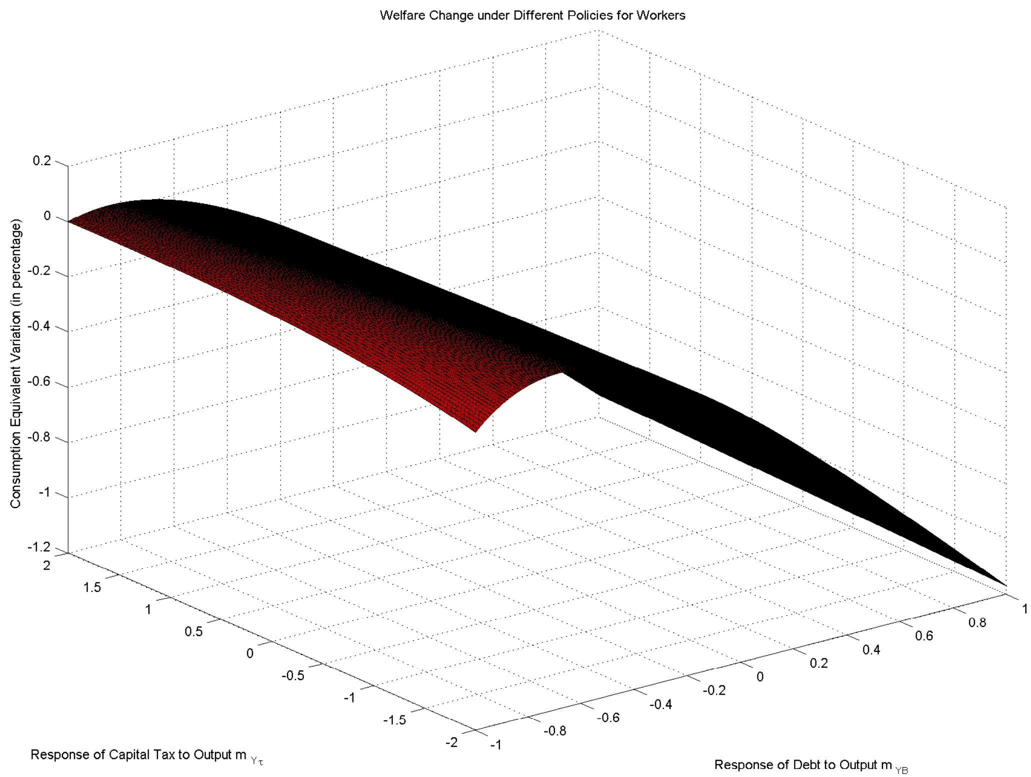
At last, the value function of workers is expressed as

$$\begin{aligned} \int_j V(x_0^j) &= \int_j \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{(c_t^j)^{1-\gamma}}{1-\gamma} \\ &= \sum_{t=0}^{\infty} \beta^t \int_j \mathbb{E}_0 \left\{\frac{(c_t^j)^{1-\gamma}}{1-\gamma}\right\} \\ &= \lambda^\gamma \sum_{t=0}^{\infty} \beta^t \mathbb{E}_0 \left\{\frac{[(1-\tau_t^n)w_t]^{1-\gamma}}{1-\gamma}\right\} \exp\left(\frac{\sigma_w^2(1-\gamma)^2}{2(1-\rho_w^2)}\right). \end{aligned} \tag{40}$$

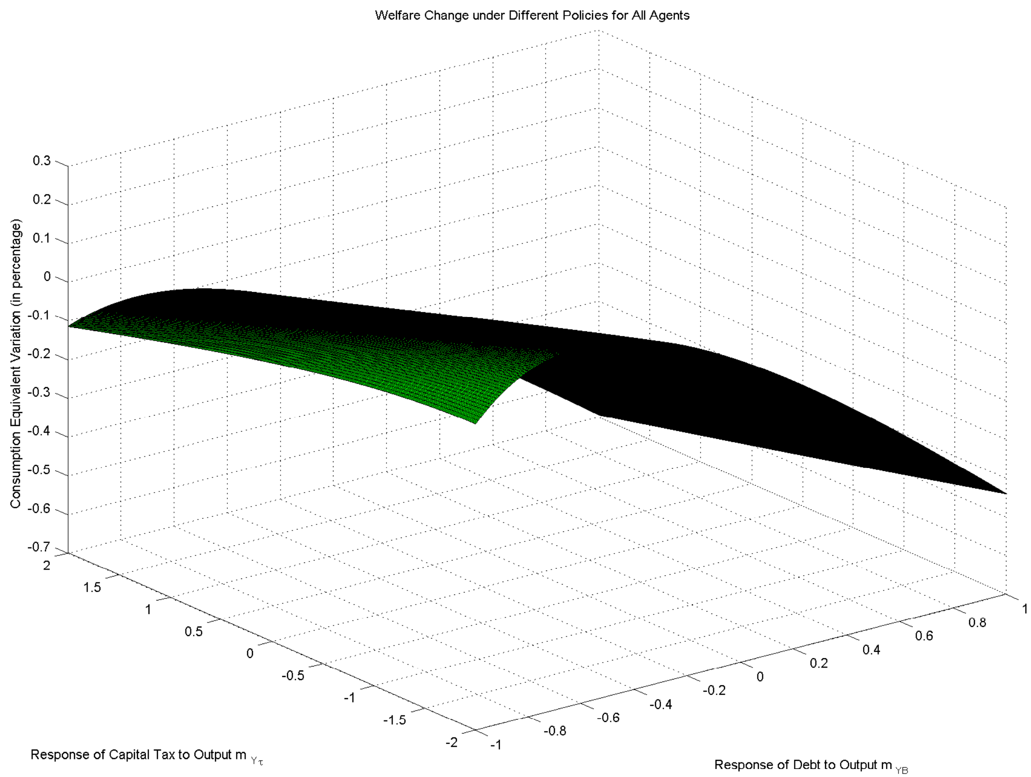
Appendix E



A1: Welfare Change Varying Capital Tax and Debt



A1: Welfare Change Varying Capital Tax and Debt (Continued)



A1: Welfare Change Varying Capital Tax and Debt (Continued)

Table 6: Comparison of Tax Policies (in Percentage)

Variables	Policy Maximizing Entrepreneurs' Welfare $m_{Y\tau} = -2$, $m_{YB} = 1$		Policy Maximizing Workers' Welfare $m_{Y\tau} = -0.06$, $m_{YB} = -0.78$		Policy Maximizing Social Welfare $m_{Y\tau} = -2$, $m_{YB} = -0.59$	
	Mean	**Std. Dev.**	**Mean**	**Std. Dev.**	**Mean**	**Std. Dev.**
Bond	23.94	20.13	-1.97	38.91	-0.81	9.02
Cap. Tax	-1.33	0.62	-0.47	-0.81	-1.42	1.23
Capital	0.06	-36.50	0.37	25.44	1.03	36.25
Entr. Con.	4.39	109.80	0.19	1.57	1.26	30.74
Wage	0.01	-21.97	0.13	8.04	0.37	9.97
Lab. Tax	0.84	0.47	0.05	0.18	0.27	0.03
Wor. Con.	-1.00	11.13	0.05	1.78	-0.01	-1.24
Welfare Change (Consumption Equivalent Variation)						
Entrepreneur	3.74		0.33		1.42	
Worker	-1.17		0.03		0.005	
Total	-0.44		0.08		0.22	

The values for all variables are percentage change compared to the baseline policy combination ($m_{Y\tau} = 0.91$, $m_{YB} = -0.62$). Cap. Tax and Lab. Tax denote the capital tax rate and the labor tax rate, respectively. Entr. Con., Wor. Con., and Ttl. Con. represent consumption for entrepreneurs, consumption for workers and consumption for total agents, respectively.